Appendix A

MATHEMATICAL FORMULAS

A.1 TRIGONOMETRIC IDENTITIES

$$\tan A = \frac{\sin A}{\cos A}, \qquad \cot A = \frac{1}{\tan A}$$

$$\sec A = \frac{1}{\cos A}, \qquad \csc A = \frac{1}{\sin A}$$

$$\sin^2 A + \cos^2 A = 1, \qquad 1 + \tan^2 A = \sec^2 A$$

$$1 + \cot^2 A = \csc^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

$$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

$$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$$

$$\sin A + \sin B = 2 \sin \frac{A + B}{2} \cos \frac{A - B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A + B}{2} \sin \frac{A - B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A + B}{2} \cos \frac{A - B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A + B}{2} \sin \frac{A - B}{2}$$

$$\cos(A \pm 90^\circ) = \mp \sin A$$

$$\sin(A \pm 90^\circ) = \pm \cos A$$

$$\tan(A \pm 90^\circ) = -\cot A$$

$$\cos(A \pm 180^\circ) = -\cos A$$

$$\sin(A \pm 180^\circ) = -\sin A$$

$$\tan(A \pm 180°) = \tan A$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan(A \pm B) = \frac{\tan A \pm B}{1 \mp \tan A \tan B}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin A = \frac{e^{jA} - e^{-jA}}{2j}, \qquad \cos A = \frac{e^{jA} + e^{-jA}}{2}$$

$$e^{jA} = \cos A + j \sin A \qquad \text{(Euler's identity)}$$

$$\pi = 3.1416$$

$$1 \text{ rad} = 57.296°$$

A.2 COMPLEX VARIABLES

A complex number may be represented as

$$z = x + jy = r\underline{/\theta} = re^{j\theta} = r(\cos\theta + j\sin\theta)$$

where $x = \text{Re}\, z = r\cos\theta, \qquad y = \text{Im}\, z = r\sin\theta$

$$r = |z| = \sqrt{x^2 + y^2}, \qquad \theta = \tan^{-1}\frac{y}{x}$$

$$j = \sqrt{-1}, \qquad \frac{1}{j} = -j, \qquad j^2 = -1$$

The complex conjugate of $z = z^* = x - jy = r\underline{/-\theta} = re^{-j\theta}$

$$= r(\cos\theta - j\sin\theta)$$

$$(e^{j\theta})^n = e^{jn\theta} = \cos n\theta + j\sin n\theta \qquad \text{(de Moivre's theorem)}$$

If $z_1 = x_1 + jy_1$ and $z_2 = x_2 + jy_2$, then $z_1 = z_2$ only if $x_1 = x_2$ and $y_1 = y_2$.

$$z_1 \pm z_2 = (x_1 + x_2) \pm j(y_1 + y_2)$$

$$z_1 z_2 = (x_1 x_2 - y_1 y_2) + j(x_1 y_2 + x_2 y_1)$$

or

$$z_1 z_2 = r_1 r_2\, e^{j(\theta_1 + \theta_2)} = r_1 r_2\underline{/\theta_1 + \theta_2}$$

$$\frac{z_1}{z_2} = \frac{(x_1 + jy_1)}{(x_2 + jy_2)} \cdot \frac{(x_2 - jy_2)}{(x_2 - jy_2)} = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + j\frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2}$$

or

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{j(\theta_1 - \theta_2)} = \frac{r_1}{r_2}\underline{/\theta_1 - \theta_2}$$

$$\sqrt{z} = \sqrt{x + jy} = \sqrt{r}\, e^{j\theta/2} = \sqrt{r}\,\underline{/\theta/2}$$

$$z^n = (x + jy)^n = r^n e^{jn\theta} = r^n \underline{/n\theta} \qquad (n = \text{integer})$$

$$z^{1/n} = (x + jy)^{1/n} = r^{1/n} e^{j\theta/n} = r^{1/n}\underline{/\theta/n + 2\pi k/n}\ (k = 0, 1, 2, \ldots, n - 1)$$

$$\ln (re^{j\theta}) = \ln r + \ln e^{j\theta} = \ln r + j\theta + j2k\pi \qquad (k = \text{integer})$$

A.3 HYPERBOLIC FUNCTIONS

$$\sinh x = \frac{e^x - e^{-x}}{2}, \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x}, \qquad \coth x = \frac{1}{\tanh x}$$

$$\operatorname{csch} x = \frac{1}{\sinh x}, \qquad \operatorname{sech} x = \frac{1}{\cosh x}$$

$$\sin jx = j \sinh x, \qquad \cos jx = \cosh x$$

$$\sinh jx = j \sin x, \qquad \cosh jx = \cos x$$

$$\sinh (x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$

$$\cosh (x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

$$\sinh (x \pm jy) = \sinh x \cos y \pm j \cosh x \sin y$$

$$\cosh (x \pm jy) = \cosh x \cos y \pm j \sinh x \sin y$$

$$\tanh (x \pm jy) = \frac{\sinh 2x}{\cosh 2x + \cos 2y} \pm j\frac{\sin 2y}{\cosh 2x + \cos 2y}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\operatorname{sech}^2 x + \tanh^2 x = 1$$

$$\sin (x \pm jy) = \sin x \cosh y \pm j \cos x \sinh y$$

$$\cos (x \pm jy) = \cos x \cosh y \mp j \sin x \sinh y$$

A.4 LOGARITHMIC IDENTITIES

$$\log xy = \log x + \log y$$

$$\log \frac{x}{y} = \log x - \log y$$

$$\log x^n = n \log x$$

$$\log_{10} x = \log x \text{ (common logarithm)}$$

$$\log_e x = \ln x \text{ (natural logarithm)}$$

If $|x| \ll 1, \ln (1 + x) \simeq x$

A.5 EXPONENTIAL IDENTITIES

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

where $e \simeq 2.7182$

$$e^x e^y = e^{x+y}$$

$$[e^x]^n = e^{nx}$$

$$\ln e^x = x$$

A.6 APPROXIMATIONS FOR SMALL QUANTITIES

If $|x| \ll 1$,

$$(1 \pm x)^n \simeq 1 \pm nx$$

$$e^x \simeq 1 + x$$

$$\ln (1 + x) \simeq x$$

$$\sin x \simeq x \quad \text{or} \quad \lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\cos \simeq 1$$

$$\tan x \simeq x$$

A.7 DERIVATIVES

If $U = U(x)$, $V = V(x)$, and a = constant,

$$\frac{d}{dx}(aU) = a\frac{dU}{dx}$$

$$\frac{d}{dx}(UV) = U\frac{dV}{dx} + V\frac{dU}{dx}$$

$$\frac{d}{dx}\left[\frac{U}{V}\right] = \frac{V\frac{dU}{dx} - U\frac{dV}{dx}}{V^2}$$

$$\frac{d}{dx}(aU^n) = naU^{n-1}$$

$$\frac{d}{dx}\log_a U = \frac{\log_a e}{U}\frac{dU}{dx}$$

$$\frac{d}{dx}\ln U = \frac{1}{U}\frac{dU}{dx}$$

$$\frac{d}{dx}a^U = d^U \ln a \frac{dU}{dx}$$

$$\frac{d}{dx}e^U = e^U\frac{dU}{dx}$$

$$\frac{d}{dx}U^V = VU^{v-1}\frac{dU}{dx} + U^V \ln U\frac{dV}{dx}$$

$$\frac{d}{dx}\sin U = \cos U\frac{dU}{dx}$$

$$\frac{d}{dx}\cos U = -\sin U\frac{dU}{dx}$$

$$\frac{d}{dx}\tan U = \sec^2 U\frac{dU}{dx}$$

$$\frac{d}{dx}\sinh U = \cosh U\frac{dU}{dx}$$

$$\frac{d}{dx}\cosh U = \sinh U\frac{dU}{dx}$$

$$\frac{d}{dx}\tanh U = \text{sech}^2 U\frac{dU}{dx}$$

A.8 INDEFINITE INTEGRALS

If $U = U(x)$, $V = V(x)$, and a = constant,

$$\int a\,dx = ax + C$$

$$\int U\,dV = UV - \int V\,dU \qquad \text{(integration by parts)}$$

$$\int U^n\,dU = \frac{U^{n+1}}{n+1} + C, \qquad n \neq -1$$

$$\int \frac{dU}{U} = \ln U + C$$

$$\int a^U\,dU = \frac{a^U}{\ln a} + C, \qquad a > 0, a \neq 1$$

$$\int e^U\,dU = e^U + C$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$$

$$\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax - 1) + C$$

$$\int x^2e^{ax}dx = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2) + C$$

$$\int \ln x\,dx = x\ln x - x + C$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax + C$$

$$\int \tan ax\,dx = \frac{1}{a}\ln \sec ax + C = -\frac{1}{a}\ln \cos ax + C$$

$$\int \sec ax\,dx = \frac{1}{a}\ln(\sec ax + \tan ax) + C$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + C$$

$$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C$$

$$\int x \sin ax\,dx = \frac{1}{a^2}(\sin ax - ax\cos ax) + C$$

$$\int x \cos ax\,dx = \frac{1}{a^2}(\cos ax + ax\sin ax) + C$$

$$\int e^{ax} \sin bx\,dx = \frac{e^{ax}}{a^2 + b^2}(a\sin bx - b\cos bx) + C$$

$$\int e^{ax} \cos bx\,dx = \frac{e^{ax}}{a^2 + b^2}(a\cos bx + b\sin bx) + C$$

$$\int \sin ax \sin bx\,dx = \frac{\sin (a-b)x}{2(a-b)} - \frac{\sin (a+b)x}{2(a+b)} + C, \qquad a^2 \neq b^2$$

$$\int \sin ax \cos bx\,dx = -\frac{\cos (a-b)x}{2(a-b)} - \frac{\cos (a+b)x}{2(a+b)} + C, \qquad a^2 \neq b^2$$

$$\int \cos ax \cos bx\,dx = \frac{\sin (a-b)x}{2(a-b)} + \frac{\sin (a+b)x}{2(a+b)} + C, \qquad a^2 \neq b^2$$

$$\int \sinh ax\,dx = \frac{1}{a}\cosh ax + C$$

$$\int \cosh ax\,dx = \frac{1}{a}\sinh ax + C$$

$$\int \tanh ax\,dx = \frac{1}{a}\ln\cosh ax + C$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

$$\int \frac{x\,dx}{x^2 + a^2} = \frac{1}{2}\ln(x^2 + a^2) + C$$

$$\int \frac{x^2\,dx}{x^2 + a^2} = x - a\tan^{-1}\frac{x}{a} + C$$

$$\int \frac{dx}{x^2 - a^2} = \begin{cases} \frac{1}{2a} \ln \frac{x-a}{x+a} + C, & x^2 > a^2 \\ \frac{1}{2a} \ln \frac{a-x}{a+x} + C, & x^2 < a^2 \end{cases}$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left(x + \sqrt{x^2 \pm a^2}\right) + C$$

$$\int \frac{xdx}{\sqrt{x^2 + a^2}} = \sqrt{x^2 + a^2} + C$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x/a^2}{\sqrt{x^2 + a^2}} + C$$

$$\int \frac{xdx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}} + C$$

$$\int \frac{x^2dx}{(x^2 + a^2)^{3/2}} = \ln\left(\frac{\sqrt{x^2 + a^2}}{a} + \frac{x}{a}\right) - \frac{x}{\sqrt{x^2 + a^2}} + C$$

$$\int \frac{dx}{(x^2 + a^2)^2} = \frac{1}{2a^2}\left(\frac{x}{x^2 + a^2} + \frac{1}{a} \tan^{-1} \frac{x}{a}\right) + C$$

A.9 DEFINITE INTEGRALS

$$\int_0^{\pi} \sin mx \sin nx \, dx = \int_0^{\pi} \cos mx \cos nx \, dx = \begin{cases} 0, & m \neq n \\ \pi/2, & m = n \end{cases}$$

$$\int_0^{\pi} \sin mx \cos nx \, dx = \begin{cases} 0, & m + n = \text{even} \\ \frac{2m}{m^2 - n^2}, & m + n = \text{odd} \end{cases}$$

$$\int_0^{2\pi} \sin mx \sin nx \, dx = \int_{-\pi}^{\pi} \sin mx \sin nx \, dx = \begin{cases} 0, & m \neq n \\ \pi, & m = n \end{cases}$$

$$\int_0^{\infty} \frac{\sin ax}{x} dx = \begin{cases} \pi/2, & a > 0, \\ 0, & a = 0 \\ -\pi/2, & a < 0 \end{cases}$$

$$\int_0^{\infty} \frac{\sin^{2x}}{x} dx = \frac{\pi}{2}$$

$$\int_0^\infty \frac{\sin^2 ax}{x^2}\,dx = |a|\,\frac{\pi}{2}$$

$$\int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^\infty e^{-ax^2}\,dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^\infty e^{-(ax^2+bx+c)}\,dx = \sqrt{\frac{\pi}{a}}\,e^{(b^2-4ac)/4a}$$

$$\int_0^\infty e^{-ax}\cos bx\,dx = \frac{a}{a^2+b^2}$$

$$\int_0^\infty e^{-ax}\sin bx\,dx = \frac{b}{a^2+b^2}$$

A.10 VECTOR IDENTITIES

If **A** and **B** are vector fields while U and V are scalar fields, then

$$\nabla(U+V) = \nabla U + \nabla V$$

$$\nabla(UV) = U\,\nabla V + V\,\nabla U$$

$$\nabla\left[\frac{U}{V}\right] = \frac{V(\nabla U) - U(\nabla V)}{V^2}$$

$$\nabla V^n = n\,V^{n-1}\,\nabla V \qquad (n = \text{integer})$$

$$\nabla(\mathbf{A}\cdot\mathbf{B}) = (\mathbf{A}\cdot\nabla)\,\mathbf{B} + (\mathbf{B}\cdot\nabla)\,\mathbf{A} + \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A})$$

$$\nabla\cdot(\mathbf{A}+\mathbf{B}) = \nabla\cdot\mathbf{A} + \nabla\cdot\mathbf{B}$$

$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$$

$$\nabla\cdot(V\mathbf{A}) = V\,\nabla\cdot\mathbf{A} + \mathbf{A}\cdot\nabla V$$

$$\nabla\cdot(\nabla V) = \nabla^2 V$$

$$\nabla\cdot(\nabla\times\mathbf{A}) = 0$$

$$\nabla\times(\mathbf{A}+\mathbf{B}) = \nabla\times\mathbf{A} + \nabla\times\mathbf{B}$$

$$\nabla\times(\mathbf{A}\times\mathbf{B}) = \mathbf{A}\,(\nabla\cdot\mathbf{B}) - \mathbf{B}\,(\nabla\cdot\mathbf{A}) + (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B}$$

$$\nabla\times(V\mathbf{A}) = \nabla V\times\mathbf{A} + V(\nabla\times\mathbf{A})$$

$$\nabla \times (\nabla V) = 0$$

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}$$

$$\oint_L \mathbf{A} \cdot d\mathbf{l} = \int_S \nabla \times \mathbf{A} \cdot d\mathbf{S}$$

$$\oint_L V d\mathbf{l} = -\int_S \nabla V \times d\mathbf{S}$$

$$\oint_S \mathbf{A} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{A}\, dv$$

$$\oint_S V d\mathbf{S} = \int_v \nabla V\, dv$$

$$\oint_S \mathbf{A} \times d\mathbf{S} = -\int_v \nabla \times \mathbf{A}\, dv$$

Appendix B

MATERIAL CONSTANTS

TABLE B.1 Approximate Conductivity* of Some Common Materials at 20°C

Material	Conductivity (siemens/meter)
Conductors	
Silver	6.1×10^7
Copper (standard annealed)	5.8×10^7
Gold	4.1×10^7
Aluminum	3.5×10^7
Tungsten	1.8×10^7
Zinc	1.7×10^7
Brass	1.1×10^7
Iron (pure)	10^7
Lead	5×10^6
Mercury	10^6
Carbon	3×10^4
Water (sea)	4
Semiconductors	
Germanium (pure)	2.2
Silicon (pure)	4.4×10^{-4}
Insulators	
Water (distilled)	10^{-4}
Earth (dry)	10^{-5}
Bakelite	10^{-10}
Paper	10^{-11}
Glass	10^{-12}
Porcelain	10^{-12}
Mica	10^{-15}
Paraffin	10^{-15}
Rubber (hard)	10^{-15}
Quartz (fused)	10^{-17}
Wax	10^{-17}

*The values vary from one published source to another due to the fact that there are many varieties of most materials and that conductivity is sensitive to temperature, moisture content, impurities, and the like.

TABLE B.2 Approximate Dielectric Constant or Relative Permittivity (ϵ_r) and Strength of Some Common Materials*

Material	Dielectric Constant ϵ_r (Dimensionless)	Dielectric Strength E(V/m)
Barium titanate	1200	7.5×10^6
Water (sea)	80	
Water (distilled)	81	
Nylon	8	
Paper	7	12×10^6
Glass	5–10	35×10^6
Mica	6	70×10^6
Porcelain	6	
Bakelite	5	20×10^6
Quartz (fused)	5	30×10^6
Rubber (hard)	3.1	25×10^6
Wood	2.5–8.0	
Polystyrene	2.55	
Polypropylene	2.25	
Paraffin	2.2	30×10^6
Petroleum oil	2.1	12×10^6
Air (1 atm.)	1	3×10^6

*The values given here are only typical; they vary from one published source to another due to different varieties of most materials and the dependence of ϵ_r on temperature, humidity, and the like.

TABLE B.3 Relative Permeability (μ_r) of Some Materials*

Material	μ_r
Diamagnetic	
Bismuth	0.999833
Mercury	0.999968
Silver	0.9999736
Lead	0.9999831
Copper	0.9999906
Water	0.9999912
Hydrogen (s.t.p.)	$\simeq$ 1.0
Paramagnetic	
Oxygen (s.t.p.)	0.999998
Air	1.00000037
Aluminum	1.000021
Tungsten	1.00008
Platinum	1.0003
Manganese	1.001
Ferromagnetic	
Cobalt	250
Nickel	600
Soft iron	5000
Silicon-iron	7000

*The values given here are only typical; they vary from one published source to another due to different varieties of most materials.

Appendix C

ANSWERS TO ODD-NUMBERED PROBLEMS

CHAPTER 1

1.1 $-0.8703\mathbf{a}_x - 0.3483\mathbf{a}_y - 0.3482\mathbf{a}_z$

1.3 (a) $5\mathbf{a}_x + 4\mathbf{a}_y + 6\mathbf{a}_z$

(b) $-5\mathbf{a}_y - 3\mathbf{a}_y + 23\mathbf{a}_z$

(c) $0.439\mathbf{a}_x - 0.11\mathbf{a}_y - 0.3293\mathbf{a}_z$

(d) $1.1667\mathbf{a}_x - 0.7084\mathbf{a}_y - 0.7084\mathbf{a}_z$

1.5 $\alpha = \dfrac{-12}{7}, \beta = \dfrac{-4}{7}$

1.7 Proof

1.9 (a) -2.8577

(b) $-0.2857\mathbf{a}_x + 0.8571\mathbf{a}_y - 0.4286\mathbf{a}_z$

(c) $65.91°$

1.11 $72.36°, 59.66°, 143.91°$

1.13 (a) $(\mathbf{B} \cdot \mathbf{A})\mathbf{A} - (\mathbf{A} \cdot \mathbf{A})\mathbf{B}$

(b) $(\mathbf{A} \cdot \mathbf{B})(\mathbf{A} \times \mathbf{A}) - (\mathbf{A} \cdot \mathbf{A})(\mathbf{A} \times \mathbf{B})$

1.15 25.72

1.17 (a) 7.681

(b) $-2\mathbf{a}_y - 5\mathbf{a}_z$

(c) $137.43°$

(d) 11.022

(e) 17.309

1.19 (a) Proof

(b) $\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2, \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2$

(c) $\left|\sin\dfrac{\theta_2 - \theta_1}{2}\right|$

1.21 (a) 10.3

(b) $-2.175\mathbf{a}_x + 1.631\mathbf{a}_y - 4.893\mathbf{a}_z$

(c) $-0.175\mathbf{a}_x + 0.631\mathbf{a}_y - 1.893\mathbf{a}_z$

CHAPTER 2

2.1 (a) $P(0.5, 0.866, 2)$
(b) $Q(0, 1, -4)$
(c) $R(-1.837, -1.061, 2.121)$
(d) $T(3.464, 2, 0)$

2.3 (a) $\rho z \cos\phi - \rho^2 \sin\phi \cos\phi + \rho z \sin\phi$
(b) $r^2(1 + \sin^2\theta \sin^2\phi + \cos\theta)$

2.5 (a) $\dfrac{1}{\rho^2 + z^2}(\rho \mathbf{a}_\rho + 4\mathbf{a}_z)$, $\left(\sin^2\theta + \dfrac{4\sin\theta}{r}\right)\mathbf{a}_r + \sin\theta\left(\cos\theta - \dfrac{4}{r}\right)\mathbf{a}_\theta$
(b) $\dfrac{\rho^2}{\sqrt{\rho^2 + z^2}}(\rho \mathbf{a}_\rho + z\mathbf{a}_z)$, $r\sin\theta\, \mathbf{a}_r$

2.7 (a) $\left(\dfrac{xyz}{x^2 + y^2} + \dfrac{xy}{\sqrt{x^2 + y^2}}\right)\mathbf{a}_x + \left(\dfrac{y^2 z}{x^2 + y^2} - \dfrac{x^2}{\sqrt{x^2 + y^2}}\right)\mathbf{a}_y + 2z\sqrt{x^2 + y^2}\,\mathbf{a}_z$
(b) $\dfrac{1}{\sqrt{x^2 + y^2}\sqrt{x^2 + y^2 + z^2}}(x\mathbf{a}_x + y\mathbf{a}_y)$

2.9 Proof

2.11 (a) $\dfrac{1}{x^2 + y^2}(x^2\mathbf{a}_x + xy\mathbf{a}_y + yz^2\mathbf{a}_z)$, 3
(b) $r(\sin^2\theta\cos\phi + r\cos^3\theta\sin\phi)\,\mathbf{a}_r + r\sin\theta\cos\theta\,(\cos\phi - r\cos\theta\sin\phi)\,\mathbf{a}_\theta$, 3

2.13 (a) $r\sin\theta\left[\sin\phi\cos\theta\,(r\sin\theta + \cos\phi)\,\mathbf{a}_r + \sin\phi\,(r\cos^2\theta - \sin\theta\cos\phi)\right.$ $\left.\mathbf{a}_\theta + 3\cos\phi\,\mathbf{a}_\phi\right]$, $5\mathbf{a}_\theta - 21.21\mathbf{a}_\phi$
(b) $\sqrt{\rho^2 + z^2}\left(\rho\mathbf{a}_\rho + \dfrac{\rho}{\sqrt{\rho^2 + z^2}}\mathbf{a}_\phi + z\mathbf{a}_z\right)$, $4.472\mathbf{a}_\rho + 0.8944\mathbf{a}_\phi + 2.236\mathbf{a}_z$

2.15 (a) An infinite line parallel to the z-axis
(b) Point $(2, -1, 10)$
(c) A circle of radius $r\sin\theta = 5$, i.e., the intersection of a cone and a sphere
(d) An infinite line parallel to the z-axis
(e) A semiinfinite line parallel to the x–y plane
(f) A semicircle of radius 5 in the x–y plane

2.17 (a) $\mathbf{a}_x - \mathbf{a}_y + 7\mathbf{a}_z$
(b) $143.26°$
(c) -8.789

2.19 (a) $-\mathbf{a}_\theta$
(b) $0.6931\mathbf{a}_\theta$
(c) $-\mathbf{a}_\theta + 0.6931\mathbf{a}_\phi$
(d) $0.6931\mathbf{a}_\phi$

2.21 (a) $3\mathbf{a}_\phi + 25\mathbf{a}_z$, $-15.6\mathbf{a}_r + 10\mathbf{a}_\phi$
(b) $2.071\mathbf{a}_\rho - 1.354\mathbf{a}_\phi + 0.4141\mathbf{a}_z$
(c) $\pm(0.5365\mathbf{a}_r - 0.1073\mathbf{a}_\theta + 0.8371\mathbf{a}_\phi)$

2.23 $(\sin\theta\cos^3\phi + 3\cos\theta\sin^2\phi)\,\mathbf{a}_r + (\cos\theta\cos^3\phi + 2\tan\theta\cos\theta\sin^2\phi -$ $\sin\theta\sin^2\phi)\,\mathbf{a}_\theta + \sin\phi\cos\phi\,(\sin\phi - \cos\phi)\,\mathbf{a}_\phi$

CHAPTER 3

3.1 (a) 2.356
(b) 0.5236
(c) 4.189
3.3 (a) 6
(b) 110
(c) 4.538
3.5 0.6667
3.7 (a) -50
(b) -39.5
3.9 $4\mathbf{a}_\rho + 1.333\mathbf{a}_z$
3.11 (a) $(-2, 0, 6.2)$
(b) $-2\mathbf{a}_x + (2.4t + 5)\mathbf{a}_z$ m/s
3.13 (a) $-0.5578\mathbf{a}_x - 0.8367\mathbf{a}_y - 3.047\mathbf{a}_z$
(b) $2.5\mathbf{a}_\rho + 2.5\mathbf{a}_\phi - 17.32\mathbf{a}_z$
(c) $-\mathbf{a}_r + 0.866\mathbf{a}_\theta$
3.15 Along $2\mathbf{a}_x + 2\mathbf{a}_y - \mathbf{a}_z$
3.17 (a) $-y^2\mathbf{a}_x + 2z\mathbf{a}_y - x^2\mathbf{a}_z, 0$
(b) $(\rho^2 - 3z^2)\mathbf{a}_\phi + 4\rho^2\mathbf{a}_z, 0$
(c) $-\frac{1}{r}\cot\theta\cos\phi + \frac{1}{r^3}\left(\frac{\cos\phi}{\sin\theta} + \cos\theta\right)\mathbf{a}_\theta, 0$
3.19 (a) Proof
(b) $2xyz$
3.21 $2(z^2 - y^2 - y)$
3.23 Proof
3.25 (a) $6yz\mathbf{a}_x + 3xy^2\mathbf{a}_y + 3x^2yz\mathbf{a}_z$
(b) $4yz\mathbf{a}_x + 3xy^2\mathbf{a}_y + 4x^2yz\mathbf{a}_z$
(c) $6xyz + 3xy^3 + 3x^2yz$
(d) $2(x^2 + y^2 + z^2)$
3.27 Proof
3.29 (a) $(6xy^2 + 2x^2 + x^5y^2)e^{xz}$, 24.46
(b) $3z(\cos\phi + \sin\phi)$, -8.1961
(c) $e^{-r}\sin\theta\cos\phi\left(1 - \frac{4}{r}\right)$, 0.8277
3.31 (a) $\frac{7}{6}$
(b) $\frac{7}{6}$
(c) Yes
3.33 50.265
3.35 (a) Proof, both sides equal 1.667
(b) Proof, both sides equal 131.57
(c) Proof, both sides equal 136.23

3.37 (a) $4\pi - 2$
(b) 7π
3.39 0
3.41 Proof
3.43 Proof
3.45 $\alpha = 1 = \beta = \gamma, -1$

CHAPTER 4

4.1 $-5.746\mathbf{a}_x - 1.642\mathbf{a}_y + 4.104\mathbf{a}_z$ mN
4.3 (a) -3.463 nC
(b) -18.7 nC
4.5 (a) 0.5 C
(b) 1.206 μC
(c) 157.9 nC
4.7 $-2.545\mathbf{a}_x + 1.054\mathbf{a}_y$ MV/m
4.9 $\dfrac{1}{2\pi} \ln \dfrac{a + \sqrt{a^2 + h^2}}{h}$
4.11 (a) Proof
(b) 0.4 mC, $31.61\mathbf{a}_z$ μV/m
4.13 $-0.591\mathbf{a}_x - 0.18\mathbf{a}_z$ N
4.15 Derivation
4.17 (a) $8.84xy\mathbf{a}_x + 8.84x^2\mathbf{a}_y$ pC/m^2
(b) $8.84y$ pC/m^3
4.19 5.357 kJ
4.21 Proof
4.23 $$D_\rho = \begin{cases} 0, & \rho < 1 \\ \dfrac{8(\rho^3 - 1)}{2\rho}, & 1 < \rho < 2 \\ \dfrac{28}{\rho} & \end{cases}$$
4.25 1050 J
4.27 (a) -1250 J
(b) -3750 nJ
(c) 0 J
(d) -8750 nJ
4.29 (a) $-2x\mathbf{a}_x - 4y\mathbf{a}_y - 8z\mathbf{a}_z$
(b) $-(x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z)\cos(x^2 + y^2 + z^2)^{1/2}$
(c) $-2\rho(z + 1)\sin\phi\,\mathbf{a}_\rho - \rho(z + 1)\cos\phi\,\mathbf{a}_\phi - \rho^2 \sin\phi\,\mathbf{a}_z$
(d) $e^{-r}\sin\theta\cos 2\phi\,\mathbf{a}_r - \dfrac{e^{-r}}{r}\cos\theta\cos 2\phi\,\mathbf{a}_\theta + \dfrac{2e^{-r}}{r}\sin 2\phi\,\mathbf{a}_\phi$
4.31 (a) $72\mathbf{a}_x + 27\mathbf{a}_y - 36\mathbf{a}_z$ V/m
(b) -30.95 pC
4.33 Proof

4.35 (a) $\dfrac{2\rho_o}{15\varepsilon_o r^2}\mathbf{a}_r, \dfrac{2\rho_o}{15\varepsilon_o r}$

(b) $\dfrac{\rho_o}{\varepsilon_o}\left(\dfrac{a^2 r}{3} - \dfrac{r^3}{5}\right)\mathbf{a}_r, \dfrac{\rho_o}{\varepsilon_o}\left(\dfrac{r^4}{20} - \dfrac{a^2 r^2}{6}\right) + \dfrac{2\,\rho_o}{15\varepsilon_o} + \dfrac{7\,\rho_o a^4}{60\varepsilon_o}$

(c) $\dfrac{8\pi\,\rho_o}{15}$

(d) Proof

4.37 (a) $-1.136\,\mathbf{a}_y$ kV/m

(b) $(\mathbf{a}_x + 0.2\mathbf{a}_y) \times 10^7$ m/s

4.39 Proof, $\dfrac{Qd}{4\pi\varepsilon_o r^3}(2\sin\theta\sin\phi\,\mathbf{a}_r - \cos\theta\sin\phi\,\mathbf{a}_\theta - \cos\phi\,\mathbf{a}_\phi)$ V/m

4.41 $\dfrac{Q^2}{8\pi\varepsilon_o a}$

4.43 6.612 nJ

CHAPTER 5

5.1 $-6.283\,A$

5.3 5.026 A

5.5 (a) $-16xyz\,\varepsilon_o$, (b) -1.131 mA

5.7 (a) 3.5×10^7 S/m, aluminum

(b) 5.66×10^6 A/m^2

5.9 (a) 0.27 mΩ

(b) 50.3 A (copper), 9.7 A (steel)

(c) 0.322 mΩ

5.11 1.000182

5.13 (a) $12.73z\mathbf{a}_z$ nC/m^2, 12.73 nC/m^3

(b) $7.427z\mathbf{a}_z$ nC/m^2, -7.472 nC/m^3

5.15 (a) $\dfrac{Q}{4\pi r^2}\left(1 - \dfrac{1}{\varepsilon_r}\right)$

(b) 0

(c) $-\dfrac{Q}{4\pi a^2}\left(1 - \dfrac{1}{\varepsilon_r}\right), \dfrac{Q}{4\pi b^2}\left(1 - \dfrac{1}{\varepsilon_r}\right)$

5.17 $-24.72\mathbf{a}_x - 32.95\mathbf{a}_y + 98.86\mathbf{a}_z$ V/m

5.19 (a) Proof

(b) $\dfrac{\rho_o a^2}{3\varepsilon_o}$

5.21 (a) $0.442\mathbf{a}_x + 0.442\mathbf{a}_y + 0.1768\mathbf{a}_z$ nC/m^2

(b) $0.2653\mathbf{a}_x + 0.5305\mathbf{a}_y + 0.7958\mathbf{a}_z$

5.23 (a) 46.23 A

(b) 45.98 μC/m^3

5.25 (a) 18.2 μs

(b) 20.58

(c) 19.23%

5.27 (a) $-1.061\mathbf{a}_x + 1.768\mathbf{a}_y + 1.547\mathbf{a}_z$ nC/m^2
(b) $-0.7958\mathbf{a}_x + 1.326\mathbf{a}_y + 1.161\mathbf{a}_z$ nC/m^2
(c) 39.79°
5.29 (a) $387.8\mathbf{a}_\rho - 452.4\mathbf{a}_\phi + 678.6\mathbf{a}_z$ V/m, $12\mathbf{a}_\rho - 14\mathbf{a}_\phi + 21\mathbf{a}_z$ nC/m^2
(b) $4\mathbf{a}_\rho - 2\mathbf{a}_\phi + 3\mathbf{a}_z$ nC/m^2, 0
(c) 12.62 mJ/m^3 for region 1 and 9.839 mJ/m^3 for region 2
5.31 (a) 705.9 V/m, 0° (glass), 6000 V/m, 0° (air)
(b) 1940.5 V/m, 84.6° (glass), 2478.6 V/m, 51.2° (air)
5.33 (a) 381.97 nC/m^2
(b) $\dfrac{0.955\mathbf{a}_r}{r^2}$ nC/m^2
(c) 12.96 μJ

CHAPTER 6

6.1 $120\mathbf{a}_x + 120\mathbf{a}_y - 12\mathbf{a}_z$, 530.52 pC/m^3
6.3 (a) $-\dfrac{\rho_o x^3}{6d\varepsilon_o} + \dfrac{\rho_o x^2}{2\varepsilon_o} + \left(\dfrac{V_o}{d} - \dfrac{\rho_o d}{3\varepsilon_o}\right)x$, $\left(\dfrac{\rho_o x^2}{2d\varepsilon_o} - \dfrac{\rho_o x}{\varepsilon_o} - \dfrac{V_o}{d} + \dfrac{\rho_o d}{3\varepsilon_o}\right)\mathbf{a}_x$
(b) $\dfrac{\rho_o d}{3} - \dfrac{\varepsilon_o V_o}{d}$, $\dfrac{\varepsilon_o V_o}{d} + \dfrac{\rho_o d}{6}$
6.5 $157.08y^4 - 942.5y^2 + 30.374$ kV
6.7 Proof
6.9 Proof
6.11 $25z$ kV, $-25\mathbf{a}_z$ kV/m, $-332\mathbf{a}_z$ nC/m^2, $\pm 332\mathbf{a}_z$ nC/m^2
6.13 9.52 V, $18.205\mathbf{a}_\rho$ V/m, $0.161\mathbf{a}_\rho$ nC/m^2
6.15 11.7 V, $-17.86\mathbf{a}_\theta$ V/m
6.17 Derivation

6.19 (a) $$-\frac{4V_o}{\pi}\sum_{n=\text{odd}}^{\infty}\frac{\sin\dfrac{n\pi x}{b}\sinh\dfrac{n\pi}{b}(a-y)}{n\sinh\dfrac{n\pi a}{b}}$$
(b) $$\frac{4V_o}{\pi}\sum_{n=\text{odd}}^{\infty}\frac{\sin\dfrac{n\pi y}{a}\sinh\dfrac{n\pi x}{a}}{n\sinh\dfrac{n\pi b}{a}}$$
(c) $$\frac{4V_o}{\pi}\sum_{n=\text{odd}}^{\infty}\frac{\sin\dfrac{n\pi y}{b}\sinh\dfrac{n\pi}{b}(a-x)}{n\sinh\dfrac{n\pi a}{b}}$$

6.21 Proof
6.23 Proof
6.25 Proof

6.27 0.5655 cm^2
6.29 Proof
6.31 (a) 100 V
(b) 99.5 nC/m^2, -99.5 nC/m^2
6.33 (a) 25 pF
(b) 63.662 nC/m^2
6.35
$$\frac{4\pi}{\dfrac{\varepsilon_1}{\frac{1}{c}-\frac{1}{d}} + +\dfrac{\varepsilon_2}{\frac{1}{b}-\frac{1}{c}} + \dfrac{\varepsilon_3}{\frac{1}{a}-\frac{1}{b}}}$$
6.37 21.85 pF
6.39 693.1 s
6.41 Proof
6.43 Proof
6.45 0.7078 mF
6.47 (a) 1 nC
(b) 5.25 nN
6.49 $-0.1891\,(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)$N
6.51 (a) $-138.24\mathbf{a}_x - 184.32\mathbf{a}_z$ V/m
(b) -1.018 nC/m^2

CHAPTER 7

7.1 (b) $0.2753\mathbf{a}_x + 0.382\mathbf{a}_y + 0.1404\mathbf{a}_z$ A/m
7.3 $0.9549\mathbf{a}_z$ A/m
7.5 (a) $28.47\,\mathbf{a}_y$ mA/m
(b) $-13\mathbf{a}_x + 13\mathbf{a}_y$ mA/m
(c) $-5.1\mathbf{a}_x + 1.7\mathbf{a}_y$ mA/m
(d) $5.1\mathbf{a}_x + 1.7\mathbf{a}_y$ mA/m
7.7 (a) $-0.6792\mathbf{a}_z$ A/m
(b) $0.1989\mathbf{a}_z$ mA/m
(c) $0.1989\mathbf{a}_x + 0.1989\mathbf{a}_y$ A/m
7.9 (a) $1.964\mathbf{a}_z$ A/m
(b) $1.78\mathbf{a}_z$ A/m
(c) $-0.1178\mathbf{a}_z$ A/m
(d) $-0.3457\mathbf{a}_x - 0.3165\mathbf{a}_y + 0.1798\mathbf{a}_z$ A/m
7.11 (a) Proof
(b) 1.78 A/m, 1.125 A/m
(c) Proof
7.13 (a) $1.36\mathbf{a}_z$ A/m
(b) $0.884\mathbf{a}_z$ A/m
7.15 (a) 69.63 A/m
(b) 36.77 A/m

7.17 (b) $H_\phi = \begin{cases} 0, & \rho < a \\ \dfrac{I}{2\pi\rho}\left(\dfrac{\rho^2 - a^2}{b^2 - a^2}\right), & a < \rho < b \\ \dfrac{I}{2\pi\rho}, & \rho > b \end{cases}$

7.19 (a) $-2\mathbf{a}_z$ A/m^2
(b) Proof, both sides equal -30 A

7.21 (a) $80\mathbf{a}_\phi$ nWb/m^2
(b) 1.756 μWb

7.23 (a) $31.43\mathbf{a}_y$ A/m
(b) $12.79\mathbf{a}_x + 6.366\mathbf{a}_y$ A/m

7.25 13.7 nWb

7.27 (a) magnetic field
(b) magnetic field
(c) magnetic field

7.29 $(14\mathbf{a}_\rho + 42\mathbf{a}_\phi) \times 10^4$ A/m, -1.011 Wb

7.31 $\dfrac{I_o\rho}{2\pi a^2}\,\mathbf{a}_\phi$

7.33 $-\dfrac{40}{\mu_o\rho^4}\,\mathbf{a}_z$ A/m^2

7.35 $\dfrac{\mu_o I}{28\pi}\left(\dfrac{\rho^2}{a^2} - 9\right) - \dfrac{8\mu_o I}{7\pi}\ln\dfrac{\rho}{3a}$

7.37 (a) 50 A
(b) -250 A

7.39 Proof

CHAPTER 8

8.1 $-4.4\mathbf{a}_x + 1.3\mathbf{a}_y + 11.4\mathbf{a}_z$ kV/m

8.3 (a) $(2, 1.933, -3.156)$
(b) 1.177 J

8.5 (a) Proof
(b) $\dfrac{2mu_o}{B_o e}$

8.7 $-86.4\mathbf{a}_z$ pN

8.9 -15.59 mJ

8.11 $1.949\mathbf{a}_x$ mN/m

8.13 $2.133\mathbf{a}_x - 0.2667\mathbf{a}_y$ Wb/m^2

8.15 (a) $-18.52\mathbf{a}_z$ mWb/m^2
(b) $-4\mathbf{a}_z$ mWb/m^2
(c) $-111\mathbf{a}_r + 78.6\mathbf{a}_\theta$ mWb/m^2

8.17 (a) 5.5
(b) $81.68\mathbf{a}_x + 204.2\mathbf{a}_y - 326.7\mathbf{a}_z$ μWb/m^2
(c) $-220\mathbf{a}_z$ A/m
(d) 9.5 mJ/m^2
8.19 476.68 kA/m
8.21 $2\dfrac{k_o}{a}\mathbf{a}_z$
8.23 (a) $25\mathbf{a}_\rho + 15\mathbf{a}_\phi - 50\mathbf{a}_z$ mWb/m^2
(b) 666.5 J/m^3, 57.7 J/m^3
8.25 $26.83\mathbf{a}_x - 30\mathbf{a}_y + 33.96\mathbf{a}_z$ A/m
8.27 (a) $-5\mathbf{a}_y$ A/m, $-6.283\mathbf{a}_y$ μWb/m^2
(b) $-35\mathbf{a}_y$ A/m, $-110\mathbf{a}_y$ μWb/m^2
(c) $5\mathbf{a}_y$ A/m, $6.283\mathbf{a}_y$ μWb/m^2
8.29 (a) 167.4
(b) 6181 kJ/m^3
8.31 11.58 mm
8.33 5103 turns
8.35 Proof
8.37 190.8 A · t, 19,080 A/m
8.39 88.5 mWb/m^2
8.41 (a) 6.66 mN
(b) 1.885 mN
8.43 Proof

CHAPTER 9

9.1 $0.4738 \sin 377t$
9.3 -54 V
9.5 (a) $-0.4t$ V
(b) $-2t^2$
9.7 9.888 μV, point A is at higher potential
9.9 0.97 mV
9.11 6A, counterclockwise
9.13 277.8 A/m^2, 77.78 A
9.15 36 GHz
9.17 (a) $\nabla \cdot \mathbf{E}_s = \rho_{vs}/\varepsilon$, $\nabla \cdot \mathbf{H}_s = 0$, $\nabla \times \mathbf{E}_s = j\omega\mu\mathbf{H}_s$, $\nabla \times \mathbf{H}_s = (\sigma - j\omega\varepsilon)\mathbf{E}_s$
(b) $\dfrac{\partial D_x}{\partial x} + \dfrac{\partial D_y}{\partial y} + \dfrac{\partial D_z}{\partial z} = \rho_v$

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{\partial B_x}{\partial t}$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{\partial B_y}{\partial t}$$
$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\partial B_z}{\partial t}$$
$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x + \frac{\partial D_x}{\partial t}$$
$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = J_y + \frac{\partial D_y}{\partial t}$$
$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_z + \frac{\partial D_z}{\partial t}$$

9.19 Proof

9.21 $-0.3z^2 \sin 10^4 t$ mC/m^3

9.23 0.833 rad/m, $100.5 \sin \beta x \sin \omega t\, \mathbf{a}_y$ V/m

9.25 (a) Yes
(b) Yes
(c) No
(d) No

9.27 $3 \cos \phi \cos (4 \times 10^6 t)\mathbf{a}_z$ A/m^2, $84.82 \cos \phi \sin (4 \times 10^6 t)\mathbf{a}_z$ kV/m

9.29 $(2 - \rho)(1 + t)e^{-\rho - t}\mathbf{a}_z$ Wb/m^2, $\dfrac{(1 + t)(3 - \rho)}{4\pi}10^7 e^{-\rho - t}\mathbf{a}_\phi$ A/m^2

9.31 (a) $6.39\underline{/242.4^\circ}$
(b) $0.2272\underline{/-202.14^\circ}$
(c) $1.387\underline{/176.8^\circ}$
(d) $0.0349\underline{/-68^\circ}$

9.33 (a) $5 \cos (\omega t - \beta x - 36.37^\circ)\mathbf{a}_y$
(b) $\dfrac{20}{\rho} \cos (\omega t - 2z)\mathbf{a}_\rho$
(c) $\dfrac{22.36}{r^2} \cos (\omega t - \phi + 63.43^\circ) \sin \theta\, \mathbf{a}_\phi$

9.35 Proof

CHAPTER 10

10.1 (a) along $\mathbf{a}_x$
(b) 1 μs, 1.047 m, 1.047×10^6 m/s
(c) see Figure C.1

10.3 (a) $5.4105 + j6.129$ /m
(b) 1.025 m
(c) 5.125×10^7 m/s
(d) $101.41\underline{/41.44^\circ}\ \Omega$
(e) $-59.16e^{-j41.44^\circ} e^{-\gamma z}\mathbf{a}_y$ mA/m

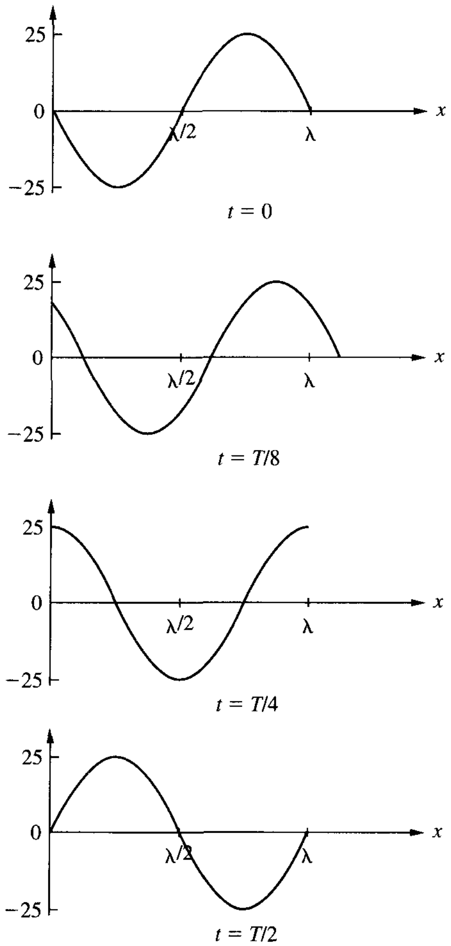

Figure C.1 For Problem 10.1.

10.5 (a) 1.732
(b) 1.234
(c) $(1.091 - j1.89) \times 10^{-11}$ F/m
(d) 0.0164 Np/m

10.7 (a) 5×10^5 m/s
(b) 5 m
(c) 0.796 m
(d) $14.05\underline{/45^\circ}\ \Omega$

10.9 (a) $0.05 + j2$ /m
(b) 3.142 m
(c) 10^8 m/s
(d) 20 m

10.11 (a) along $-x$-direction
(b) 7.162×10^{-10} F/m
(c) $1.074 \sin(2 \times 10^8 + 6x)\mathbf{a}_z$ V/m

10.13 (a) lossless
(b) 12.83 rad/m, 0.49 m
(c) 25.66 rad
(d) 4617 Ω
10.15 Proof
10.17 5.76, $-0.2546 \sin(10^9 t - 8x)\mathbf{a}_y + 0.3183 \cos(10^9 t - 8x)\mathbf{a}_z$ A/m
10.19 (a) No
(b) No
(c) Yes
10.21 2.183 m, 3.927×10^7 m/s
10.23 0.1203 mm, 0.126 Ω
10.25 2.94×10^{-6} m
10.27 (a) 131.6 Ω
(b) $0.1184 \cos^2(2\pi \times 10^8 t - 6x)\mathbf{a}_x$ W/m^2
(c) 0.3535 W
10.29 (a) 2.828×10^8 rad/s, $\dfrac{0.225}{\rho} \sin(\omega t - 2z)\mathbf{a}_\phi$ A/m
(b) $\dfrac{9}{\rho^2} \sin^2(\omega t - 2z)\mathbf{a}_z$ W/m^2
(c) 11.46 W
10.31 (a) $-\dfrac{1}{3}, \dfrac{2}{3}, 2$
(b) $-10 \cos(\omega t + z)\mathbf{a}_x$ V/m, $26.53 \cos(\omega t + z)\mathbf{a}_y$ mA/m
10.33 26.038×10^{-6} H/m
10.35 (a) 0.5×10^8 rad/m
(b) 2
(c) $-26.53 \cos(0.5 \times 10^8 t + z)\mathbf{a}_x$ mA/m
(d) $1.061\mathbf{a}_z$ W/m^2
10.37 (a) 6.283 m, 3×10^8 rad/s, $7.32 \cos(\omega t - z)\mathbf{a}_y$ V/m
(b) $-0.0265 \cos(\omega t - z)\mathbf{a}_x$ A/m
(c) -0.268, 0.732
(d) $\mathbf{E}_1 = 10 \cos(\omega t - z)\mathbf{a}_y - 2.68 \cos(\omega t + z)\mathbf{a}_y$ V/m,
$\mathbf{E}_2 = 7.32 \cos(\omega t - z)\mathbf{a}_y$ V/m, $P_{1ave} = 0.1231\mathbf{a}_z$ W/m^2,
$P_{2ave} = 0.1231\mathbf{a}_z$ W/m^2
10.39 See Figure C.2.
10.41 Proof, $\mathbf{H}_s = \dfrac{-j20}{\omega\mu_o}\left[k_y \sin(k_x x) \sin(k_y y)\mathbf{a}_x + k_x \cos(k_x x) \cos(k_y y)\mathbf{a}_y\right]$
10.43 (a) 36.87°
(b) $79.58\mathbf{a}_y + 106.1\mathbf{a}_z$ mW/m^2
(c) $(-1.518\mathbf{a}_y + 2.024\mathbf{a}_z) \sin(\omega t + 4y - 3z)$ V/m, $(1.877\mathbf{a}_y - 5.968\mathbf{a}_y)$ $\sin(\omega t - 9.539y - 3z)$ V/m
10.45 (a) 15×10^8 rad/s
(b) $(-8\mathbf{a}_x + 6\mathbf{a}_y - 5\mathbf{a}_z) \sin(15 \times 10^8 t + 3x + 4y)$ V/m

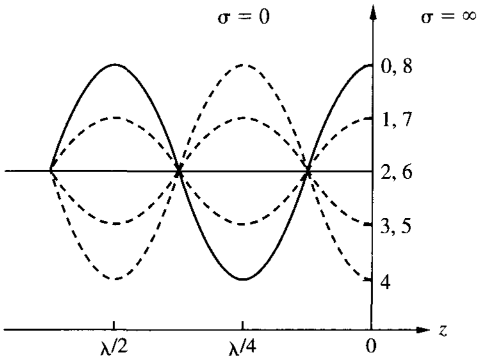

Figure C.2 For Problem 10.39; curve n corresponds to $t = nT/8$, $n = 0, 1, 2, \ldots$

CHAPTER 11

11.1 0.0104 Ω/m, 50.26 nH/m, 221 pF/m, 0 S/m
11.3 Proof
11.5 (a) $13.34\underline{/-36.24^\circ}$, 2.148×10^7 m/s
(b) 1.606 m
11.7 Proof
11.9 $\dfrac{V_o}{Z_o} \sin(\omega t - \beta z)$ A
11.11 (a) Proof
(b) (i) $\dfrac{2n}{n+1}$
(ii) 2
(iii) 0
(iv) 1
11.13 798.3 rad/m, 3.542×10^7 m/s
11.15 Proof
11.17 (a) 0.4112, 2.397
(b) $34.63\underline{/-40.65^\circ}$ Ω
11.19 $0.2\underline{/40^\circ}$ A
11.21 (a) 46.87 Ω
(b) 48.39 V
11.23 Proof
11.25 $10.2 + j13.8$ Ω, $0.7222\underline{/154^\circ}$, 6.2
11.27 (a) $j300$ Ω
(b) $15 + j0.75$ Ω
11.29 $0.35 + j0.24$
11.31 (a) 125 MHz
(b) $72 + j72$ Ω
(c) $0.444\underline{/120^\circ}$
11.33 (a) $35 + j34$ Ω,
(b) 0.375λ

11.35 (a) 24.5 Ω,
(b) 55.33 Ω, 67.74 Ω

11.37 10.25 W

11.39 $20 + j15$ mS, $-j10$ mS, $-6.408 + j5.189$ mS, $20 + j15$ mS, $j10$ mS, $2.461 + j5.691$ mS

11.41 (a) $34.2 + j41.4\ \Omega$
(b) 0.38λ, 0.473λ,
(c) 2.65

11.43 4, $0.6\angle{-90°}$, $27.6 - j52.8\ \Omega$

11.45 2.11, 1.764 GHz, $0.357\angle{-44.5°}$, $70 - j40\ \Omega$

11.47 See Figure C.3.

11.49 See Figure C.4.

11.51 (a) 77.77 Ω, 1.8
(b) 0.223 dB/m, 4.974 dB/m
(c) 3.848 m

11.53 $9.112\ \Omega < Z_o < 21.03\ \Omega$

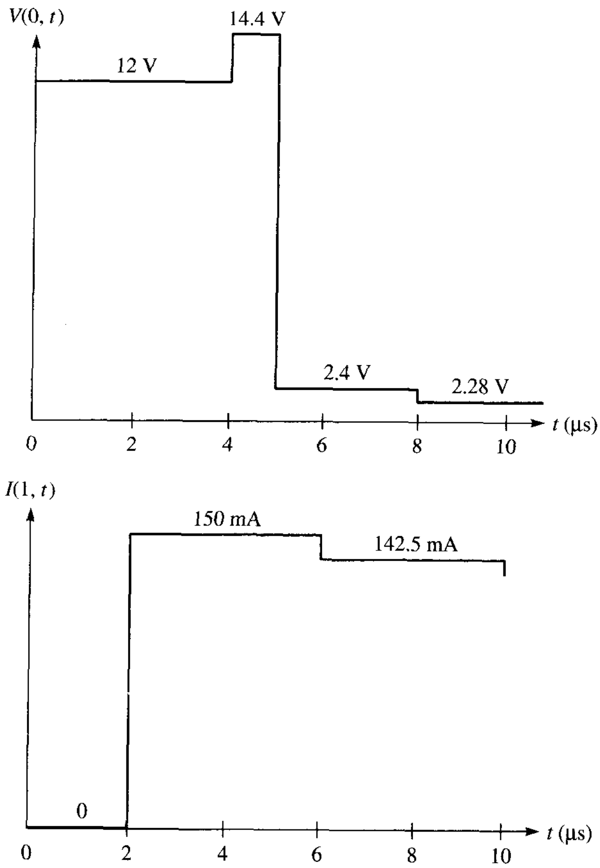

Figure C.3 For Problem 11.47.

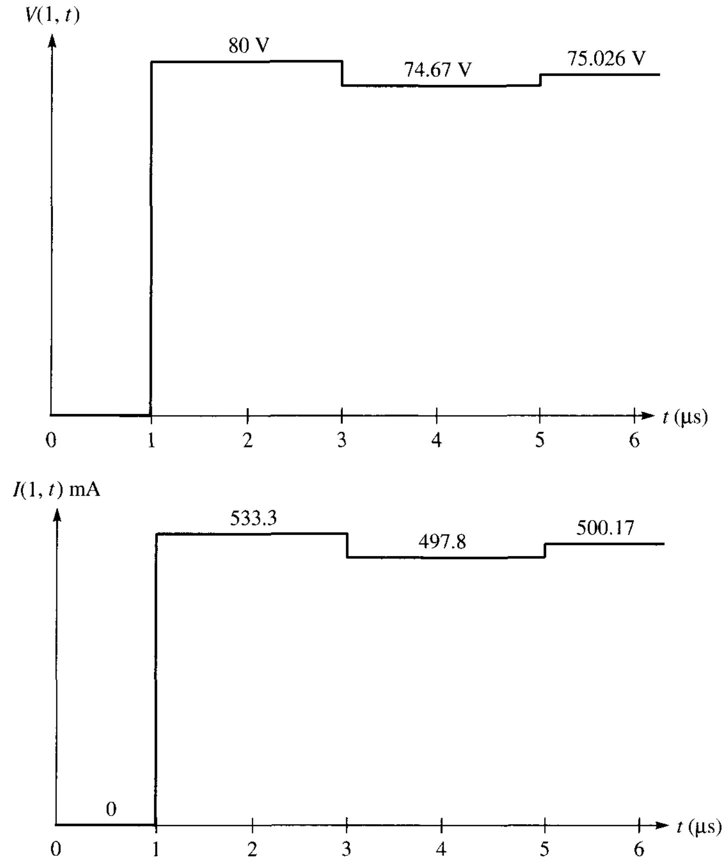

Figure C.4 For Problem 11.49.

CHAPTER 12

12.1 Proof

12.3 (a) See Table C.1
(b) $\eta_{TE13} = 573.83\ \Omega$, $\eta_{TM15} = 3.058\ \Omega$
(c) 3.096×10^7 m/s

12.5 (a) No
(b) Yes

12.7 430 ns

12.9 375.1 Ω, 0.8347 W

12.11 (a) TE_{23}
(b) $j400.7$/m
(c) 985.3 Ω

12.13 (a) Proof
(b) 4.06×10^8 m/s, 2.023 cm, 5.669×10^8 m/s, 2.834 cm

TABLE C.1

Mode	f_c (GHz)
TE_{01}	0.8333
TE_{10}, TE_{02}	1.667
TE_{11}, TM_{11}	1.863
TE_{12}, TM_{12}	2.357
TE_{03}	2.5
TE_{13}, TM_{13}	3
TE_{04}	3.333
TE_{14}, TM_{14}	3.727
TE_{05}, TE_{23}, TM_{23}	4.167
TE_{15}, TM_{15}	4.488

12.15 (a) 1.193
(b) 0.8381

12.17 4.917

12.19 $\dfrac{\omega^2\mu^2 b^3 a}{4\pi^2\eta} H_o^2 \sin^2 \dfrac{\pi y}{b}\,\mathbf{a}_z,\ \dfrac{\omega^2\mu^2 a b^3 H_o}{4\pi^2\eta}$

12.21 0.04637 Np/m, 4.811 m

12.23 (a) 2.165×10^{-2} Np/m
(b) 4.818×10^{-3} Np/m

12.25 Proof

12.27 Proof, $-\dfrac{1}{h^2}\left(\dfrac{m\pi}{a}\right)\left(\dfrac{p\pi}{c}\right) H_o \sin\left(\dfrac{m\pi x}{a}\right)\cos\left(\dfrac{n\pi y}{b}\right)\cos\left(\dfrac{p\pi z}{c}\right)$

12.29 (a) TE_{011}
(b) TM_{110}
(c) TE_{101}

12.31 See Table C.2

TABLE C.2

Mode	f_r (GHz)
011	1.9
110	3.535
101	3.333
102	3.8
120	4.472
022	3.8

12.33 (a) 6.629 GHz
(b) 6,387

12.35 $2.5\,(-\sin 30\pi x \cos 30\pi y\mathbf{a}_x + \cos 30\pi x \sin 30\pi y\mathbf{a}_y) \sin 6 \times 10^9$

CHAPTER 13

13.1 $-\dfrac{50\eta\beta}{\mu r}\sin(\omega t - \beta r)(-\sin\phi\mathbf{a}_\phi + \cos\theta\cos\phi\mathbf{a}_\theta)$ V/m

$-\dfrac{50\beta}{\mu r}\sin(\omega t - \beta r)(\sin\phi\mathbf{a}_\theta + \cos\theta\cos\phi\mathbf{a}_\phi)$ A/m

13.3 94.25 mV/m, j0.25 mA/m

13.5 1.974 Ω

13.7 28.47 A

13.9 (a) $E_{\theta s} = \dfrac{j\eta I_o\beta\ell e^{-j\beta r}\sin\theta}{8\pi r}$, $H_{\phi s} = \eta E_{\theta s}$

(b) 1.5

13.11 (a) 0.9071 μA

(b) 25 nW

13.13 See Figure C.5

13.15 See Figure C.6

13.17 $8\sin\theta\cos\phi$, 8

13.19 (a) $1.5\sin\theta$

(b) 1.5

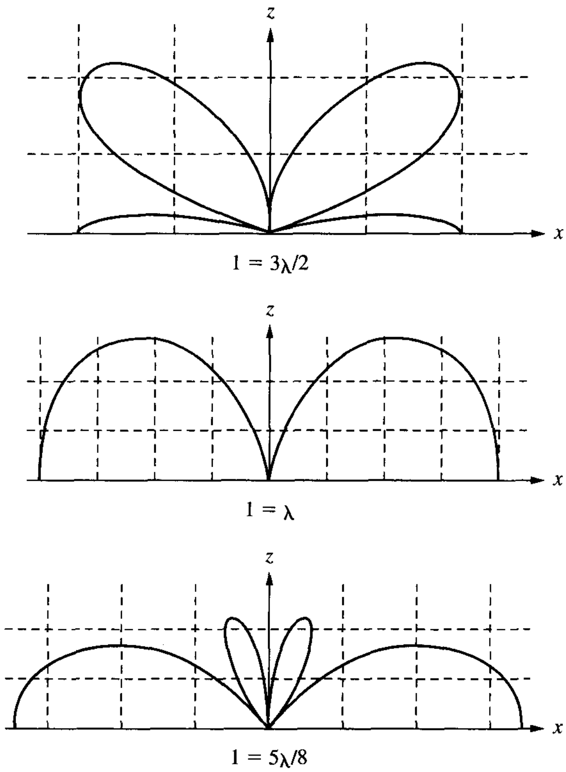

Figure C.5 For Problem 13.13.

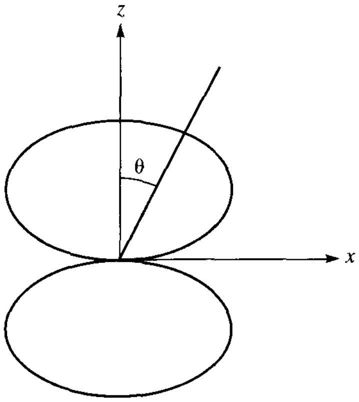

Figure C.6 For Problem 13.15.

(c) $\dfrac{1.5\lambda^2 \sin^2\theta}{4\pi}$

(d) 3.084 Ω

13.21 99.97%

13.23 (a) $1.5 \sin^2\theta$, 5

(b) $6 \sin^2\theta \cos^2\phi$, 6

(c) $66.05 \cos^2\theta \sin^2 \phi/2$, 66.05

13.25 $\dfrac{j\eta\beta I_o d\ell}{2\pi r} \sin\theta \cos\left(\dfrac{1}{2}\beta d \cos\theta\right)\mathbf{a}_\theta$

13.27 See Figure C.7

13.29 See Figure C.8

13.31 0.2686

13.33 (a) Proof

(b) 12.8 μW

13.35 21.28 pW

13.37 19 dB

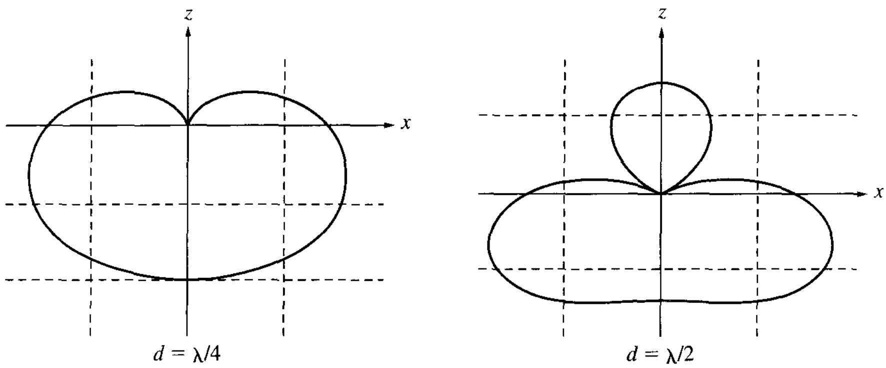

Figure C.7 For Problem 13.27.

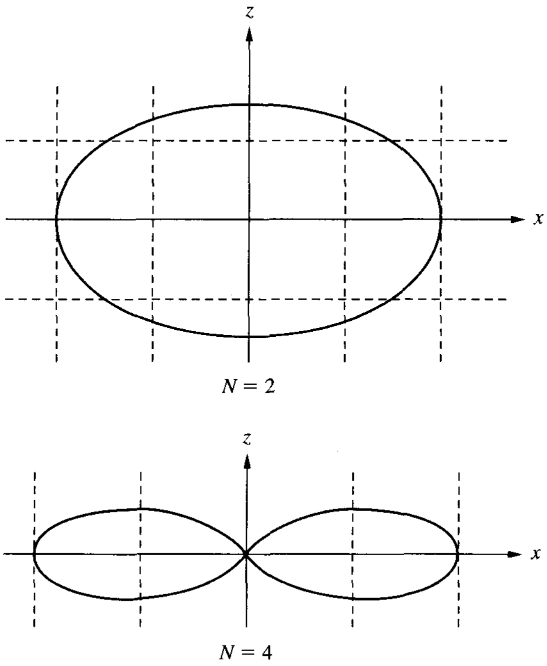

Figure C.8 For Problem 13.29.

13.39 (a) 1.708 V/m
(b) 11.36 μV/m
(c) 30.95 mW
(d) 1.91 pW
13.41 77.52 W

CHAPTER 14

14.1 Discussion
14.3 $0.33 - j0.15, 0.5571 - j0.626$
14.5 3.571
14.7 Proof
14.9 1.428
14.11 (a) 0.2271
(b) 13.13°
(c) 376
14.13 (a) 29.23°
(b) 63.1%
14.15 $\alpha_{10} = 8686\alpha_{14}$
14.17 Discussion

CHAPTER 15

15.1 See Figure C.9
15.3 (a) 10.117, 1.56
(b) 10.113, 1.506
15.5 Proof
15.7 6 V, 14 V
15.9 $V_1 = V_2 = 37.5$, $V_3 = V_4 = 12.5$

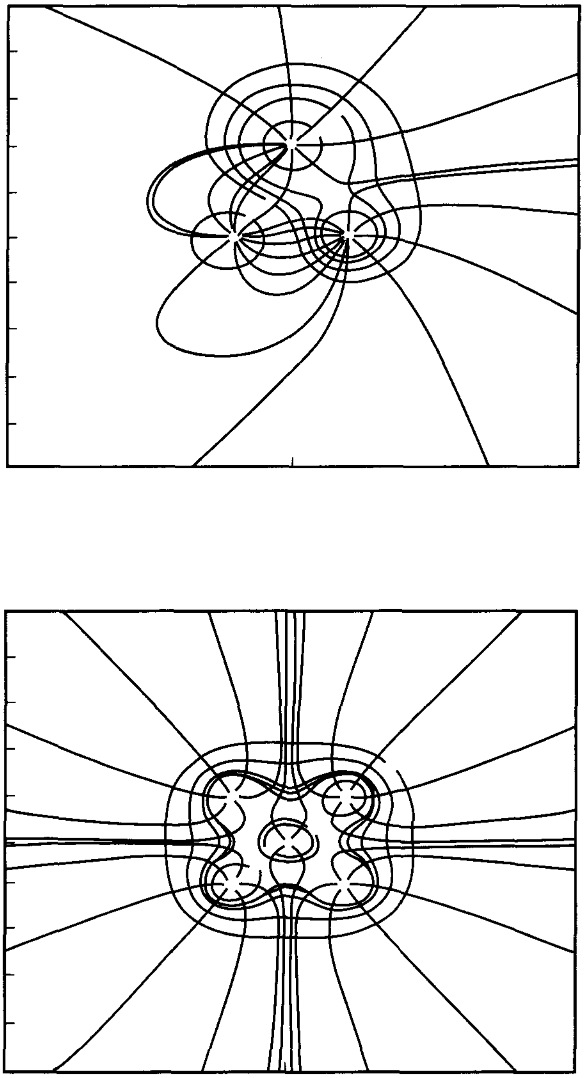

Figure C.9 For Problem 15.1.

15.11 (a) Matrix $[A]$ remains the same, but $-h^2 \rho_s/\varepsilon$ must be added to each term of matrix $[B]$.

(b) $V_a = 4.276$, $V_b = 9.577$, $V_c = 11.126$
$V_d = -2.013$, $V_e = 2.919$, $V_f = 6.069$
$V_g = -3.424$, $V_h = -0.109$, $V_i = 2.909$

15.13 Numerical result agrees completely with the exact solution, e.g., for $t = 0$, $V(0, 0) = 0$, $V(0.1, 0) = 0.3090$, $V(0.2, 0) = 0.5878$, $V(0.3, 0) = 0.809$, $V(0.4, 0) = 0.9511$, $V(0.5, 0) = 1.0$, $V(0.6, 0) = 0.9511$, etc.

15.15 12.77 pF/m (numerical), 12.12 pF/m (exact)

15.17 See Table C.3

TABLE C.3

θ (degrees)	C (pF)
10	8.5483
20	9.0677
30	8.893
40	8.606
⋮	⋮
170	11.32
180	8.6278

15.19 (a) Exact: $C = 80.26$ pF/m, $Z_o = 41.56\ \Omega$; for numerical solution, see Table C.4

TABLE C.4

N	C (pF/m)	Z_o (Ω)
10	82.386	40.486
20	80.966	41.197
40	80.438	41.467
100	80.025	41.562

(b) For numerical results, see Table C.5

TABLE C.5

N	C (pF/m)	Z_o (Ω)
10	109.51	30.458
20	108.71	30.681
40	108.27	30.807
100	107.93	30.905

15.21 Proof

15.23 (a) At (1.5, 0.5) along 12 and (0.9286, 0.9286) along 13.
(b) 56.67 V

15.25
$$\begin{bmatrix} 0.8788 & -0.208 & 0 & -0.6708 \\ -2.08 & 1.528 & -1.2 & -0.1248 \\ 0 & -1.2 & 1.408 & -0.208 \\ -0.6708 & -0.1248 & -0.208 & 1.0036 \end{bmatrix}$$

15.27 18 V, 20 V

15.29 See Table C.6

TABLE C.6

Node No.	FEM	Exact
8	4.546	4.366
9	7.197	7.017
10	7.197	7.017
11	4.546	4.366
14	10.98	10.66
15	17.05	16.84
16	17.05	16.84
17	10.98	10.60
20	22.35	21.78
21	32.95	33.16
22	32.95	33.16
23	22.35	21.78
26	45.45	45.63
27	59.49	60.60
28	59.49	60.60
29	45.45	45.63

15.31 Proof