3

Fundamental Properties of Antennas

The beginning of wisdom is the definition of terms.
—Socrates

An incredible diversity of radiating structures can be found in modern communication and radar systems. Ignoring superficial geometrical or mechanical differences, what distinguishes these structures from each other? In later theoretical work we will find it convenient to classify antennas according to the analytical methods used, and/or the physical basis for the radiation. But in a practical sense, the most important distinguishing characteristics are those that describe the performance of the antenna in a real system. Our goal in this chapter will be to define several important parameters representing the radiation characteristics in transmission and reception, as well as the circuit properties of antennas, and show how these can be computed from a field analysis. This "black-box" description of the antenna will provide well-defined goals for later theoretical work, as well as providing the necessary tools for using antennas in practical systems.

3.1 TRANSMITTING PROPERTIES

In a communications system, the antenna is a transducer that couples energy from one electronic system to another at some distant location. Two important aspects of the system are immediately apparent: 1) the antenna-field interaction, and 2) the antenna-circuit interaction. In the first case we are concerned with the directive properties of the antenna and the polarization; that is, we must determine in which direction and how effectively the antenna interacts with electromagnetic waves. In the second case we require the circuit properties of the antenna when used as both a transmitter or receiver; for example, we must know the equivalent impedance of an antenna as perceived by the generator, or the Thevenin equivalent circuit when used as a receiver. We will begin by considering the directive properties of the antenna.

3.1.1 Radiation Patterns and Directivity

A useful idealization in antenna theory is the concept of an *isotropic radiator*, which is a fictitious point source that radiates uniformly in all directions. For this case the Poynting's vector $\overline{\mathcal{P}}$

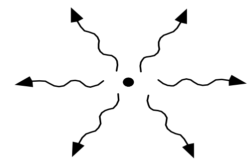


Figure 3.1 Isotropic source.

should have only a radial component and depend only on the radial distance from the source, *i.e.* $\overline{\mathcal{P}} = \mathcal{P}_r(r)\hat{r}$. The total average power radiated, P_{rad}, is found by integrating the power flux through a sphere of radius r around the source, giving

$$P_{\text{rad}} = \oiint \overline{\mathcal{P}} \cdot d\overline{S} = \oiint \left(\overline{\mathcal{P}} \cdot \hat{r}\right) r^2 \, d\Omega = 4\pi r^2 \mathcal{P}_r(r) \tag{3.1}$$

or

$$\overline{\mathcal{P}}(r) = \frac{P_{\text{rad}}}{4\pi r^2}\hat{r} \qquad \text{(isotropic source)} \tag{3.2}$$

Since the Poynting vector is related to the fields through

$$\overline{\mathcal{P}} = \frac{1}{2}\text{Re}\left\{\overline{E} \times \overline{H}^*\right\}, \tag{3.3}$$

then the radiation fields (those that are responsible for carrying real power away from the antenna) must vary as

$$\overline{E}, \overline{H} \propto \frac{e^{-jkr}}{r} \tag{3.4}$$

which is the form of an outward spherical propagating wave. This behavior was discussed in Chapter 2 in connection with the Sommerfeld radiation conditions, but here we see it as a simple consequence of energy conservation. The Sommerfeld radiation conditions also tell us that, far enough away from the source, the radiated fields are transverse to the direction of propagation (TEM waves), so in spherical coordinates we expect

$$\begin{aligned} \overline{E} &\Rightarrow E_\theta\hat{\theta} + E_\phi\hat{\phi} \\ \overline{H} &\Rightarrow H_\theta\hat{\theta} + H_\phi\hat{\phi} = \frac{-E_\phi}{\eta}\hat{\theta} + \frac{E_\theta}{\eta}\hat{\phi} \end{aligned} \tag{3.5}$$

The fields close to the antenna—the "near-fields"—can be quite complicated and not described well by the simple spherical wave (3.4), but the radiation condition assures us that in the "far-field" we always have a transverse wave with $1/r$ dependence.

In reality, no physically realizable source can radiate uniformly in all directions—there is *always* some angular dependence to the radiation. We will prove this later. So for real antennas we must modify (3.4) and write the far-fields as

$$\begin{aligned} \overline{E} &= \frac{e^{-jkr}}{r}\overline{f}(\theta, \phi) \\ \overline{H} &= \frac{1}{\eta}\hat{r} \times \overline{E} \end{aligned} \qquad \text{(far-field)} \tag{3.6}$$

where $\overline{f}(\theta, \phi)$ is the far-field *radiation pattern* or *scattering function*. The vector nature of $\overline{f}$ is necessary to describe the polarization properties of the fields. The far-field Poynting's vector for a real antenna then takes the form

$$\overline{\mathcal{P}} \propto \hat{r}\frac{U_0}{r^2}\left|f(\theta, \phi)\right|^2 \tag{3.7}$$

where $\left|f(\theta, \phi)\right|^2$ is referred to as the *power* or *intensity* pattern. Unless otherwise stated, radiation pattern plots are usually of the power-patterns and not the field patterns. (But remember, on a dB scale the two are exactly the same since $10\log|f(\theta,\phi)|^2 = 20\log|f(\theta,\phi)|$.) Furthermore, the pattern functions are often normalized so that the maximum value of $|f(\theta,\phi)|$ is 1, in which case U_0 is the *maximum radiation intensity*.

A plot of the pattern function conveys much information about the directive properties of the antenna. As an example, fig. 3.2 shows part of a radiation pattern for an electrically small dipole antenna oriented along the z-axis. Such plots also provide a nice visual method for comparing

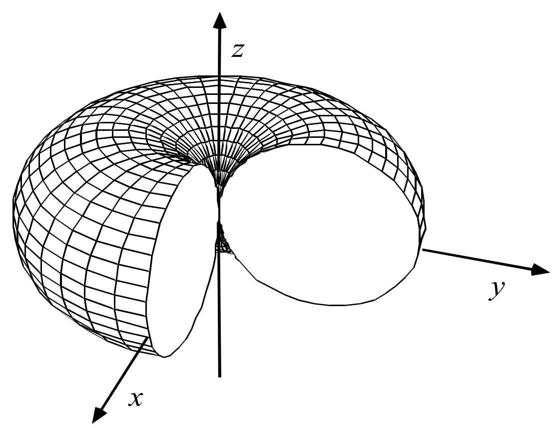


Figure 3.2 3D radiation pattern for a dipole antenna, $f(\theta, \phi) = \sin\theta$, cut to show cross section.

antennas. Comparisons of the magnitude of power density in various directions are most meaningful when the two antennas radiate the same total power. For example, a plot of the relative power density patterns (not normalized) in the x-z plane for a $\hat{z}$-directed point dipole antenna alongside that of an isotropic radiator *for the same total radiated power* is shown in figure 3.3. In this

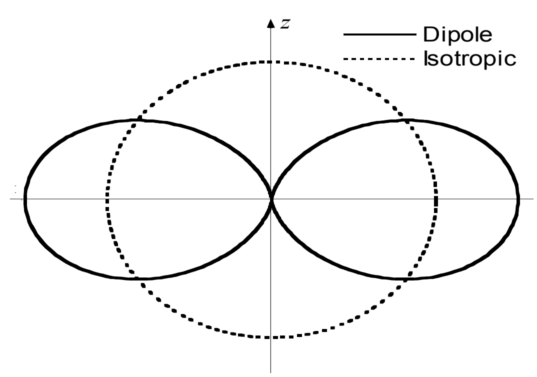


Figure 3.3 Comparison of isotropic source and point dipole power patterns.

case we see that in some directions the radiation intensity in the far-field is greater if a dipole is used than if an isotropic source were used, and in some directions it is smaller. This increase or decrease in power over that of an isotropic source is called the "directive gain", or "directivity" of an antenna. It is *not* a power gain in the sense of an active amplifier. The directive gain is only a function of the angular direction, so it can be represented as $D(\theta, \phi)$, and is defined quantitatively so that the power density is given by

$$\mathcal{P}_r = \left(\frac{P_{\mathrm{rad}}}{4\pi r^2}\right) D(\theta, \phi). \tag{3.8}$$

An isotropic source therefore has a directivity of $D(\theta, \phi) = 1$. Re-arranging this expression gives

$$D(\theta, \phi) = 4\pi r^2 \frac{\mathcal{P}_r}{\oiint \overline{\mathcal{P}} \cdot d\overline{S}} \tag{3.9}$$

or, in terms of the pattern function from (3.7),

$$D(\theta, \phi) = 4\pi \frac{|f(\theta, \phi)|^2}{\oiint |f(\theta, \phi)|^2 \, d\Omega} \tag{3.10}$$

In practice the maximum value of the directivity function is most often used, and in fact this maximum value is often casually referred to as **THE** directivity. If the pattern function has been normalized, the *maximum direcivity* D_0 is

$$D_0 = \frac{1}{\frac{1}{4\pi} \oiint |f(\theta, \phi)|^2 \, d\Omega} \qquad \text{so that} \qquad D(\theta, \phi) = D_0 \, |f(\theta, \phi)|^2 \tag{3.11}$$

As we will see, the directivity is an extremely important parameter in practice. However, a graphical representation of the pattern function $f(\theta, \phi)$ is also quite useful, and can be drawn in a number of ways to enhance or suppress various features of the function as desired. The accompanying ***Mathematica*** file `PlottingExamples.nb` includes a summary of many of these formats, and example code for generating radiation patterns from a given $f(\theta, \phi)$.

Two common representations of a highly directive beam are shown in fig. 3.4. While there are many examples of patterns not represented by this figure (including the dipole pattern of fig. 3.2), it serves to help us define several common terms that are used to describe radiation patterns in general. The central feature of importance is the *main beam* or *main lobe* which describes the direction and angular extent of the bulk of the radiated energy. Usually there is a single main lobe as shown, along with a number of much smaller minor lobes, or *side lobes* which usually account for a small fraction of the total radiated energy. The minor lobes immediately adjacent to the main lobe are called the *first sidelobe*. Of special importance in many applications is the *sidelobe level*, which refers to the peak sidelobe power relative to the peak main lobe power. Many radiation patterns also involve deep pattern *nulls* in one or more directions, which are a result of almost perfect destructive interference of the radiation from all parts of the antenna.

The main lobe is characterized by two commonly used (and fairly self-explanatory) beamwidth parameters: (a) the *Half-Power Beam Width* (HPBW), which is the angular width between half power (-3dB) directions around the main beam (also sometimes referred to as the full-width-half-maximum (FWHM) beamwidth); and (b) the *First-Null Beam Width* (FNBW) which is the angular width between the null locations on either side of the main beam location, assuming there are nulls there. Clearly either of these definitions are dependent on the cross section of the pattern

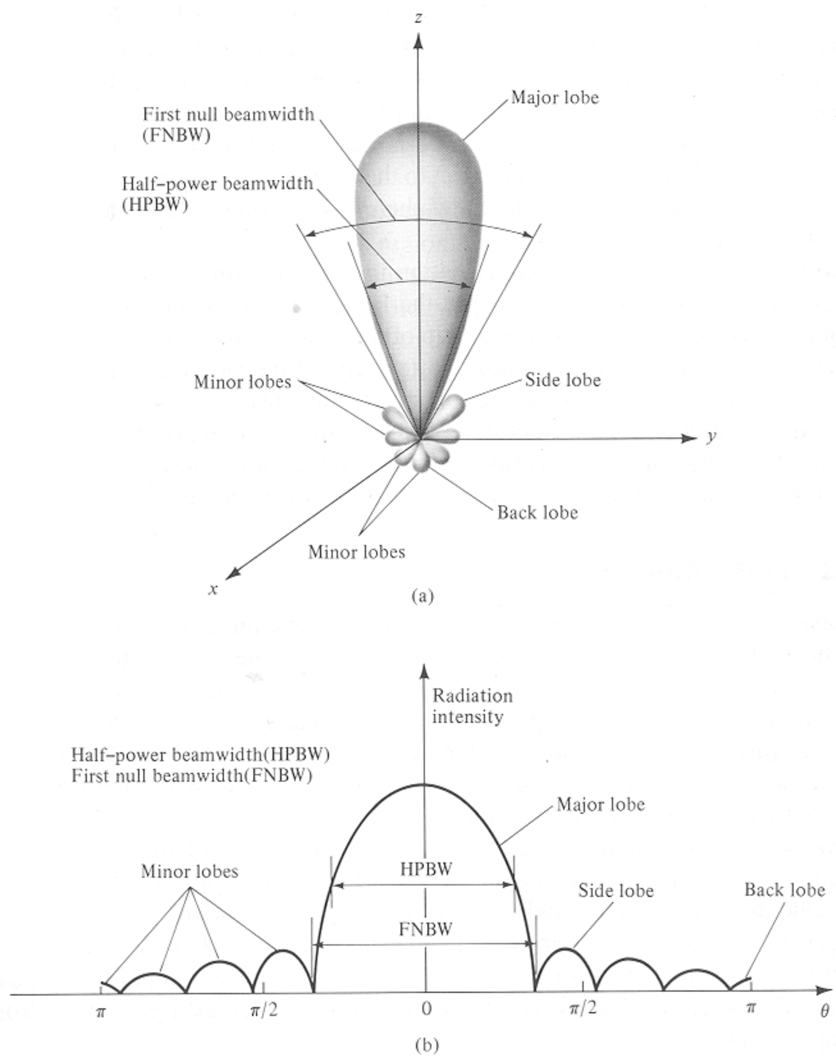


Figure 3.4 Two views of a typical highly directive radiation pattern displaying a single main lobe and several smaller sidelobes, and illustrating beam-width definitions and sidelobe terminology. (a) Three-dimensional view. (b) Two dimensional cross section.

used. For highly directivity antennas with a single narrow lobe or beam (sometimes called a "pencil-beam"), it is common to define a equivalent *beam solid angle* as shown in figure 3.5. The beam is replaced by a cone of solid angle Ω_A over which all of the radiation is distributed uniformly. Therefore the power density at the end of the cone at a distance r is given by

$$\mathcal{P} = \frac{P_{\text{rad}}}{\Omega_A r^2}. \tag{3.12}$$

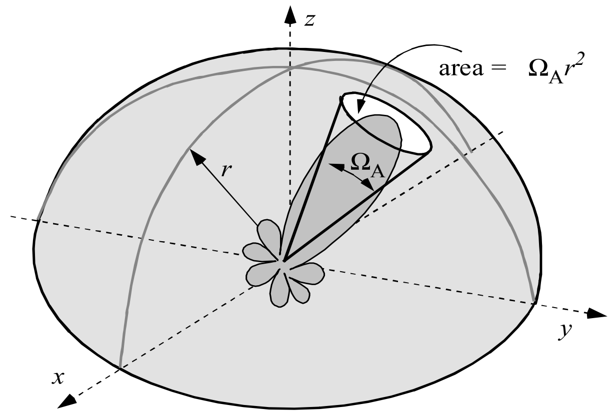


Figure 3.5 Beam solid angle.

Comparing this with (3.8) we can relate the beam-sold angle to the directivity as

$$\Omega_A = \frac{4\pi}{D_0} \qquad \text{or} \qquad D_0 = \frac{4\pi}{\Omega_A}. \tag{3.13}$$

The narrower the beam, the higher the directivity. In terms of the pattern function,

$$\Omega_A = \oint\!\!\oint |f(\theta,\phi)|^2 \, d\Omega \tag{3.14}$$

As such, we can compute a beam solid angle for any pattern, not simply pencil-beam patterns. The ratio $\Omega_A/4\pi$ describes the fraction of visible space into which radiation can be found.

Show how beam-solid-angle can be approximately found by multiplying angles in the orthogonal planes.

Example: *Beam solid angle and directivity of a Hertzian (point) dipole*

The normalized field pattern of a Hertzian dipole is

$$f(\theta,\phi) = \sin\theta$$

From (3.14) we get

$$\Omega_A = \oint\!\!\oint |f(\theta,\phi)|^2 \sin\theta \, d\theta d\phi = 2\pi \int_0^\pi \sin^3\theta \, d\theta = \frac{8\pi}{3}$$

And so the maximum directivity is, from (3.13),

$$D_0 = \frac{4\pi}{8\pi/3} = \frac{3}{2} = 1.5$$

In terrestrial communications links and radar it is common (and quite natural) to specify the angles θ and ϕ relative to the ground as angles of *elevation* and *azimuth*, respectively. These are described in fig. 3.6 for a typical ground-based reflector antenna system.

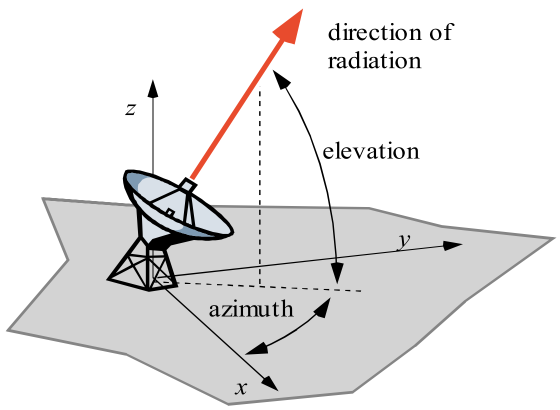


Figure 3.6 Angles are often specified in terms of azimuth and elevation in terrestrial antennas.

3.1.2 Application-specific radiation patterns

There are numerous other terms used to describe radiation patterns in special cases. For example, radio broadcasting antennas are usually designed for uniform "coverage" over all angles in azimuth, as shown in fig. 3.7a. This type of antenna is called *omni-directional*. Note that omni-directional antennas are *not* isotropic radiators; they do not (and can not) radiate uniformly in elevation. We may anticipate that omni-directional antennas will have azimuthal (cylindrical) symmetry in their physical construction. For example, a vertical cylindrical tower or pole can have omni-directional characteristics. Since the signals are transmitted to ground-based receivers, an ideal short-range broadcasting antenna would in fact have a narrow beam cross section in elevation—as narrow as possible—to achieve high gain and hence maximum transmitting range. In contrast, point-to-point communication links have highly-directional beams as shown in fig. 3.7b.

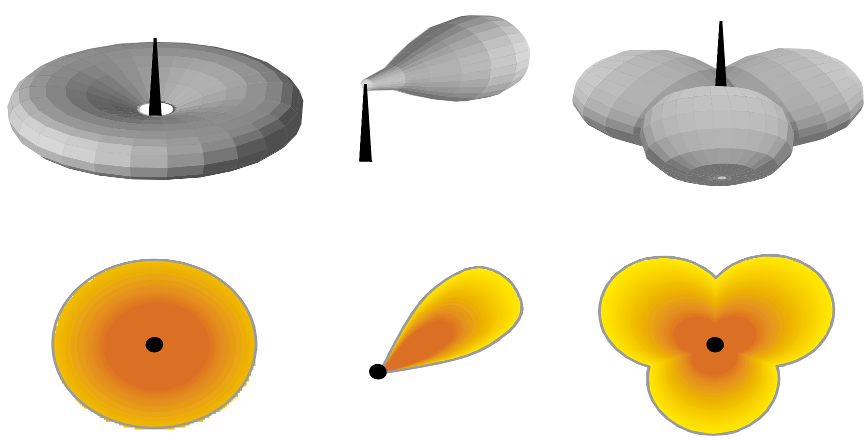


Figure 3.7 Omnidirectional, directional, and sector coverage antennas.

Surveilance or air-traffic-control radars may require the high gain of a directional beam but the full azimuthal coverage of an omni-directional antenna; this is often achieved by mechanical rotation of the beam, which is sometimes referred to as a *spotlight* beam. We will also discuss how to achieve beam-motion electronically when we discuss phased-arrays in a later chapter. Full coverage in azimuth can also be obtained using several *sector-coverage* antennas as in fig. 3.7c. In this case, each antenna has a high gain in elevation, and covers a specific "sector" of space in azimuth. In this case the designer must balance the improved gain resulting from limited sector-coverage with the complexity and economics of using multiple antennas; often 3-4 antennas prove optimum.

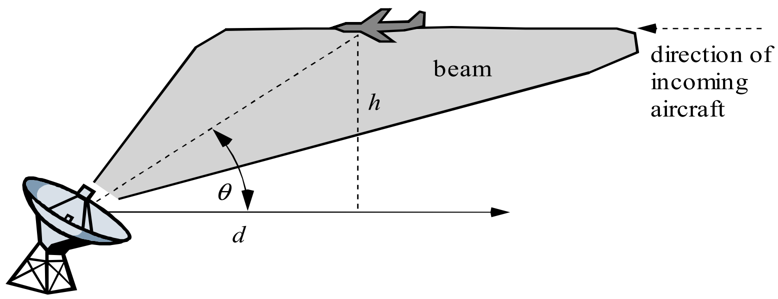


Figure 3.8 Cosecant pattern.

In some cases the radiation patterns of the antennas are designed for a specific shape in elevation. The classic example of a *shaped-beam* antenna is the so-called "cosecant" antenna which is used in air-traffic-control radar. The cosecant pattern maintains (over a limited range of angle in elevation) a constant power density independent at a fixed height h from the ground, independent of the distance d from the antenna. This can be easily seen using the geometry of fig. 3.9,

$$\mathcal{P} = \frac{P_t D_0}{4\pi R^2}|f(\theta)|^2 = \frac{P_t D_0}{4\pi}\frac{\csc^2\theta}{d^2+h^2} = \frac{P_t D_0}{4\pi h}$$

since $\csc\theta = (d^2+h^2)/h$. As we will later see, this is particularly useful in radar systems. However, shaped beam antennas are not confined to radar applications or cosecant patterns. For example, many modern communications satellites now employ shaped-beams to selectively illuminate portions of the the globe.

Show HNS spaceway for shaped-beam directed at various parts of Earth.

Narrow beams are often used in radar and communications applications, and can be realized using large apertures or arrays. In these applications, narrow (highly directive) beams with very low sidelobes are required, but as we will find later, there is an important tradeoff between sidelobe level and beamwidth. The optimum balance between the two is very application specific, and numerous designs are in use. One specific type of radar that deserves special mention is the *monopulse* radar, which requires an antenna that can switch between two narrow beams as shown in fig.X, called sum and difference patterns. These beam patterns are used for precise angle tracking. The target is first acquired using the sum pattern, so that is it known to fall somewhere near the pattern peak. The antenna is then switched to the difference mode. Unless the target is located exactly in the central null of the difference pattern, a signal will be detected in the receiver. The signal will have a different sign depending on which central lobe the target falls in. The angular position of the antenna is then adjusted until this signal is minimized. Difference patterns with very deep nulls are required for precise angle tracking.

Figure 3.9 Monopulse (a) sum and (b) difference patterns.

3.1.3 Radiation Efficiency, Impedance Mismatch, and Gain

There are practical complications regarding our definition of directivity that are important in many situations. In a real antenna the radiated power comes from a generator connected via some feed network to the antenna terminals, as shown in figure 3.10. Ideally all of the average available power from the generator, $P_{\text{avail}} = V_g^2/8\text{Re}\{Z_g\}$ (V_g is a peak quantity), would be radiated by the antenna, but this is never the case due to impedance mismatching and ohmic losses in the antenna. Let's ignore matching losses for the moment and consider just the effect of antenna losses: if an

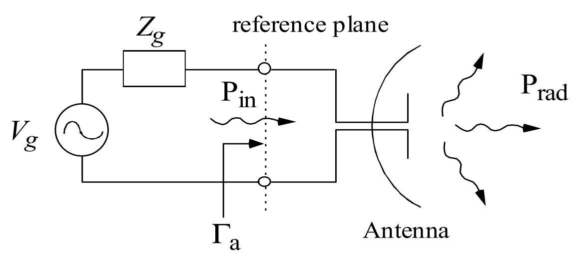


Figure 3.10 A transmitting system with the source network replaced by a Thevenin equivalent.

amount of power P_{in} is delivered to the antenna terminals and only some fraction, e_r, is radiated, then

$$P_{\text{rad}} = e_r P_{\text{in}} \tag{3.15}$$

and e_r is called the *radiation efficiency* of the antenna. It can be expressed as

$$e_r = \frac{P_{\text{rad}}}{P_{\text{in}}} = \frac{P_{\text{rad}}}{P_{\text{rad}} + P_{\text{lost}}} \tag{3.16}$$

The lost power could be due to ohmic losses or other mechanisms (such as excitation of trapped modes in the dielectric substrate of a planar antenna) that prevent energy from being radiated in the desired fashion.

Substituting (3.15) into (3.8) gives

$$\mathcal{P}_r(\theta, \phi) = \frac{P_{\text{in}}}{4\pi r^2} \left[e_r D(\theta, \phi) \right] = \frac{P_{\text{in}}}{4\pi r^2} G(\theta, \phi). \tag{3.17}$$

The quantity in brackets is called the *antenna gain*, denoted by $G(\theta, \phi)$. It is the quantity of most practical interest, but unfortunately the term has been abused in the literature so that directivity and gain are now often used interchangeably. For efficient antennas where $e_r \approx 1$ then little harm is done by this abuse of words, but it should always be remembered that there is a significant difference between the gain and directivity of the antenna.

Matching losses can be treated in much the same way, but in this case we do not include the mismatch factor in the antenna gain function, since it is not an intrinsic limitation of the antenna but rather the way it is being fed. With a mismatch between the source and antenna, (3.15) is modified as

$$P_{\mathrm{rad}} = e_r(1 - |\Gamma_a|^2)P_{\mathrm{avail}} \tag{3.18}$$

where Γ_a is the reflection coefficient defined by

$$\Gamma_a = \frac{Z_{\mathrm{ant}} - Z_g}{Z_{\mathrm{ant}} + Z_g} \tag{3.19}$$

and Z_{ant} is the antenna terminal impedance. The far-field power density is then written as

$$\mathcal{P}_r(\theta,\phi) = \frac{P_{\mathrm{avail}}}{4\pi r^2}(1 - |\Gamma_a|^2)e_r D(\theta,\phi) = \frac{P_{\mathrm{avail}}}{4\pi r^2}(1 - |\Gamma_a|^2)G(\theta,\phi). \tag{3.20}$$

Expressions like (3.20) are frequently used to predict the frequency response of an antenna system, in which case it should be remembered that the antenna impedance *and* gain are both functions of frequency, although we have not shown this dependence explicitly in our notation.

3.1.4 Polarization Properties

In the far-field, the electric field in general has both a $\hat{\theta}$ and $\hat{\phi}$ components, as in (3.5). The relative magnitude and phasing of these components determine the *polarization* of the antenna, which describes the vector orientation of the radiated field and how that orientation varies in time. If we take the phase reference as the $\hat{\theta}$-component, we can write

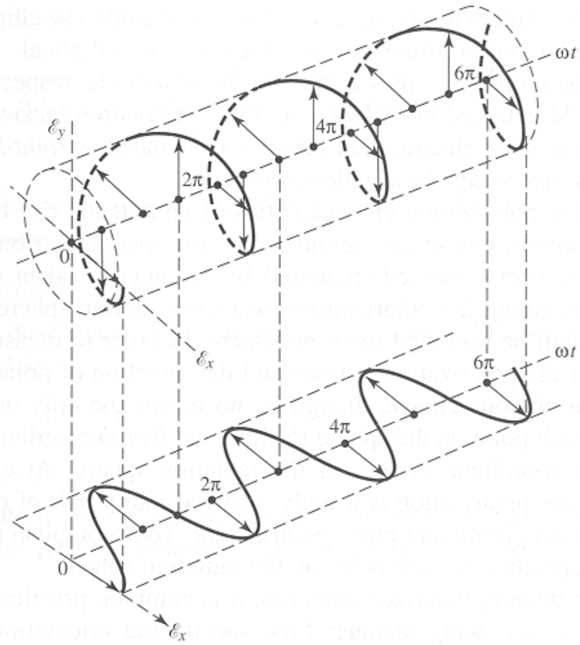
$$\overline{E} = |E_\theta|\hat{\theta} + |E_\phi|e^{\jmath\Delta\psi}\hat{\phi}$$

where $\Delta\psi$ is the relative phase diference between the two components.

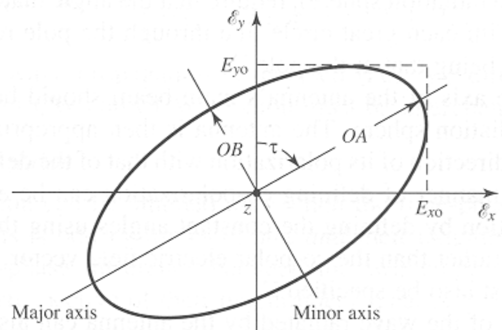
The simplest case to consider is a *linearly polarized* field, which is described by either of the following two conditions: (a) one of the two field components is zero (such as the fields of the Hertzian dipoles when the dipole moment is coincident with the $\hat{z}$-axis); or (b) the relative phase is $\Delta\psi = 0$ or π. In each of these cases, the tip of the field vector traces out a line on the $\hat{\theta}$-$\hat{\phi}$ surface.

Another special case is the *circularly polarized* field, when occurs for the very special case of $|E_\theta| = |E_\phi|$ and $\Delta\psi = \pm\pi/2$. In this case the tip of the field vector rotates around the axis of propagation, as depicted in fig. 3.11a, tracing out a circle on the $\hat{\theta}$-$\hat{\phi}$ surface. The direction of rotation is determined by the sign of $\Delta\psi$, and is described in two ways. The wave is said to have *right-hand polarization*, or RHP, if the field rotation is in the direction of the fingers of the right hand when the thumb is pointing in the direction of propagation. Otherwise it is called *left-handed* circular polarization, or LHP. Equivalently, RHP is sometimes referred to as *clockwise* or CW polarization, since the field is observed to follow a clockwise rotation it when looking along the direction of propagation, and LHP is similarly called counter-clockwise, or CCW polarization.

When both field components are present with different magnitudes, and/or a relative phase that is not one of the special cases described above, the field vector then traces out an ellipse as shown in fig. 3.11b. We define the *axial ratio* as the ratio of major to minor axes on the



(a) Rotation of wave



(b) Polarization ellipse

Figure 3.11 (a) Locus of the field vector along the direction of propagation for a RHP (or CW) circularly polarized wave (b) General polarization ellipse.

polarization ellipse. Therefore a cicularly polarized signal will have an axial ratio of unity, and a linearly-polarized signal will have an axial ratio of infinity (ideally).

In general we will represent the field polarization using our pattern function vector $\overline{f}(\theta, \phi)$, which we can write as

$$\overline{f}(\theta, \phi) = f_\theta(\theta, \phi)\hat{\theta} + f_\phi(\theta, \phi)$$

This notation emphasizes that the polarization properties of the antenna are dependent on the direction of observation; each field component has its own independent radiation pattern. Alternatively we can write the pattern vector in terms of the vectors long the major and minor axes of the polarization ellipse,

$$\overline{f}(\theta,\phi) = f_p(\theta,\phi)\hat{p} + f_q(\theta,\phi)\hat{q} \tag{3.21}$$

in which case the axial ratio is simply

$$AR \equiv \frac{f_p}{f_q} \tag{3.22}$$

For directional antennas, we are primarily interesed in the polarization-properties in the direction of maximum radiation. When an antenna is said to transmit/receive a certain polarization, it is implied that the radiation is transmitted/received along this direction. However it is rarely the case that the antenna will maintain the same polarization or axial ratio over all possible angles. In fig. **??** we show examples of measured radiation patterns in the two orthogonal polarizations for circular and linearly-polarized antennas. It is clear that the desired characteristics in each case are maintained over a relatively small range of angles around $\theta = 0°$.

Show plots of fp and fq for linear and circular antennas.

Show a radiation pattern that is measured with a rotating dipole.3

Any state of polarization can be decomposed into a combination of two orthogonal polarizations. We have already done the obvious cases of decompsition along the $\hat{\theta}$-$\hat{\phi}$ or $\hat{p}$-$\hat{q}$ directions, but an wave can also be decomposed into a combination of RHP and LHP waves. For example, we can write (3.21) as

$$\overline{f} = A(\underbrace{\hat{p} + \jmath\hat{q}}_{LHP}) + B(\underbrace{\hat{p} - \jmath\hat{q}}_{RHP}) \tag{3.23}$$

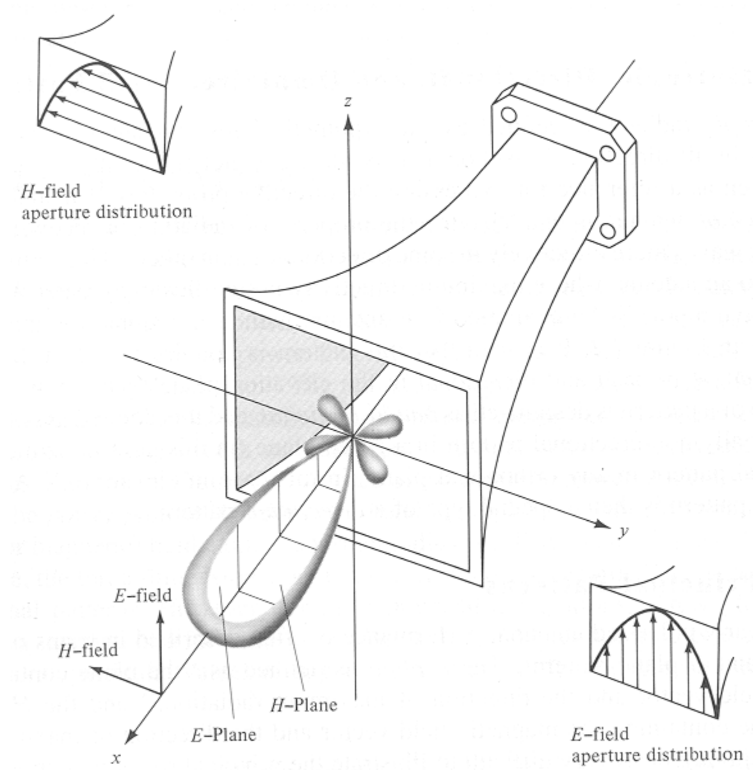
where $A = (f_p - \jmath f_q)/2$ and $B = (f_p + \jmath f_q)/2$

Most communications or radar systems are designed to operate in linear or circular polarization. However it is difficult to obtain perfect linear or circular polarization. When the polarization characteristics differ from the desired characteristics, the undesired polarization is called the *cross-polarization*. Therefore in the antenna of fig. **??**b, we woudl consider the x-component to be the cross-polarized radiation. For a circularly-polarized antenna producing a desired rotation, the cross-polarization represents a coupling to the opposite sense of rotation, but this is hard to measure and hence the axial ratio is specified instead, as determined from measurements like that of fig. **??**.

We have already pointed out that three-dimensional radiation patterns are often represented by two orthogonal cross sections. For linearly polarized antennas, these cross-sections are commonly chosen based on the orientation of fields produced by the antenna. A waveguide horn antenna, shown in fig. 3.11, serves as a good example. If the horn is fed by a rectangular waveguide operated in the dominant mode, the radiated field will have the same linear polarization as the waveguide mode, which is the $\hat{x}$-direction in fig. 3.11. The plane defined by the beam direction ($\hat{z}$) and the electric field ($\hat{x}$) is then referred to as the E-plane, and the cross section of the beam taken along this plane is called the *E-plane pattern*. The *H-plane pattern* is similarly defined. Clearly it is only possible to define an E- or H-plane pattern for a linearly-polarized antenna.

3.1.5 Terminal Impedance and Bandwidth

From the point of view of the electronic system to which the antenna is connected, the antenna is simply another circuit element with a complex impedance or admittance that must be matched to the rest of the network for efficient power transfer. Nearly all antennas encountered in practice have a complicated frequency-dependence of the input impedance, which limits the bandwidth of operation when connected to a generator with a different internal impedance. Some traveling wave antennas and self-complementary structures that are physically large compared with a wavelength



can have a wide range of operating frequencies over which the circuit characteristics are relatively constant, but in general, smaller antennas support a standing wave of current and consequently display multiple resonance characteristics. Most often the antenna will be used in a limited range of frequencies around a well defined center frequency. In this case, the antenna impedance can often be adequately modeled by a simple series or parallel RLC circuit. The choice of model is dictated by the nature of the resonance. Figure 3.12 shows the frequency dependence of the impedance and admittance of a center-fed dipole antenna, which is representative of many resonant antenna structures. The frequency dependence of the reactance is characterized by alternating poles and zeroes. When the real (resistive) part of the impedance is relatively flat near resonance and the imaginary part (reactance) has a positive linear slope, then a series RLC model is appropriate. If the admittance displays this variation (flat conductance, positive linear slope for susceptance), then a parallel RLC circuit is appropriate.

The equivalent circuit parameters near resonance can be determined directly from the plots. For a series resonator we write

$$Z = R + \jmath\omega L + \frac{1}{\jmath\omega C} = R + jX \tag{3.24}$$

Resonance is defined as the point where the reactance is zero, $X = 0$, which occurs at the resonant frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{3.25}$$

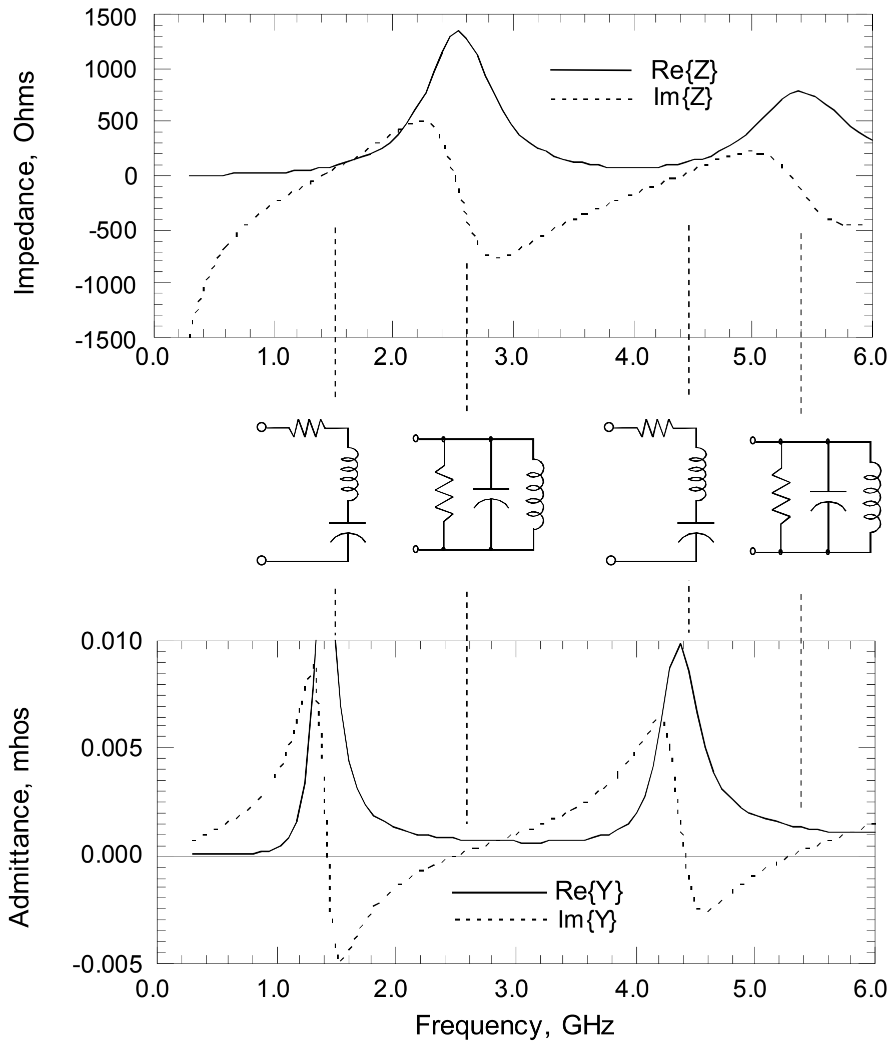


Figure 3.12 Typical impedance-versus-frequency plot (top graph) for an antenna (in this case a center-fed dipole) and corresponding admittance (bottom graph). The appropriate circuit models are identified near resonance points.

Near resonance we can write $\omega = \omega_0 + \Delta\omega$, and we find

$$Z(\omega) \approx R + j(2\Delta\omega L) \tag{3.26}$$

so $\partial X/\partial\omega \approx 2L$ near resonance; in other words, the slope of the reactance and the zero crossing determine the equivalent reactive elements. The real part R is easily read from the graph directly.

The unloaded Q-factor of a series resonator is

$$Q = \frac{\omega_0 L}{R} \cong \frac{\omega_0}{2R}\frac{\partial X}{\partial \omega}\bigg|_{\omega=\omega_0} \tag{3.27}$$

and the impedance is sometimes written in terms of this quantity as

$$Z \approx R\left(1 + j2Q\frac{\Delta\omega}{\omega_0}\right) \tag{3.28}$$

We can characterize the bandwidth of the antenna in a communications circuit by connecting it directly to a generator with a source resistance of R_g, as shown in figure 3.13. Note that the additional loss in the circuit will lower the overall Q-factor of the system. This net Q-factor is called the *loaded Q-factor*, Q_L, and is given by

$$Q_L = \frac{\omega_0 L}{R_g + R} \tag{3.29}$$

Assuming the generator is matched to the antenna at resonance ($R_g = R$), the radiated power is

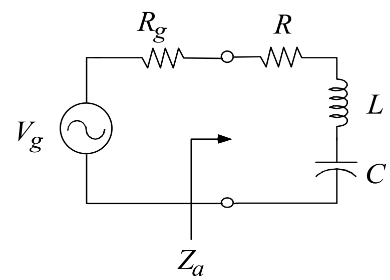


Figure 3.13 Series resonant antenna connected to generator.

$$P_{\text{rad}} = \frac{1}{2}R\left|\frac{V_g}{2R\left(1 + jQ\Delta\omega/\omega_0\right)}\right|^2 = \frac{|V_g|^2/8R}{1 + Q^2\left(\Delta\omega/\omega_0\right)^2} \tag{3.30}$$

The half-power (3 dB) bandwidth, BW, is then found as

$$\text{BW} = \frac{2\omega_0}{Q} = \frac{\omega_0}{Q_L} \approx \left[\frac{4R}{\partial X/\partial\omega}\right]_{\omega=\omega_0} \tag{3.31}$$

(note this is in unit of [rad/s]). This simply confirms that a broadband antenna should have the smallest possible variation in reactance with frequency.

In electronic systems it is common to specify bandwidth as a fraction of the carrier or center frequency. The fractional bandwidth, FBW, is defined as

$$\text{FBW} = \frac{\text{BW}}{\omega_0} \times 100\% \tag{3.32}$$

A typical communications system may have a fractional bandwidth of 5-10%, which requires an antenna with $Q < 20$.

At high frequencies the antenna is usually fed from a generator with a 50 Ω source impedance, and for maximum power transfer over a broad frequency range, a matching network (fig. 3.14) must be used. In principle the matching network can be designed to increase the inherent bandwidth of the antenna as described above, but there are both theoretical and practical limitations on such matching networks when the load is reactive. The design of impedance matching networks is well beyond the scope of this book, but is an important aspect of antenna system design, and the interested reader is referred to the encyclopedic work by Matthaei [**?**] for details.

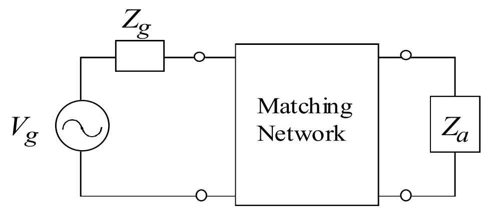


Figure 3.14 Impedance matching.

3.2 RECEIVING PROPERTIES OF ANTENNAS

3.2.1 Relationship Between Transmitting and Receiving Properties

For antennas comprised of linear reciprocal conducting or dielectric materials we can establish a general relationship between the transmitting and receiving properties using the Lorentz reciprocity theorem. To do this we consider a typical communications system shown below in fig. 3.15a. The antennas and the relative separation and orientation are completely arbitrary. The same physical situation is shown in fig. 3.15b, except that the transmitter and receiver circuits have been interchanged. We assume that the transmitter and receiver are both connected to the antennas through a length of single-mode transmission line or waveguide. In each case we assume that the only

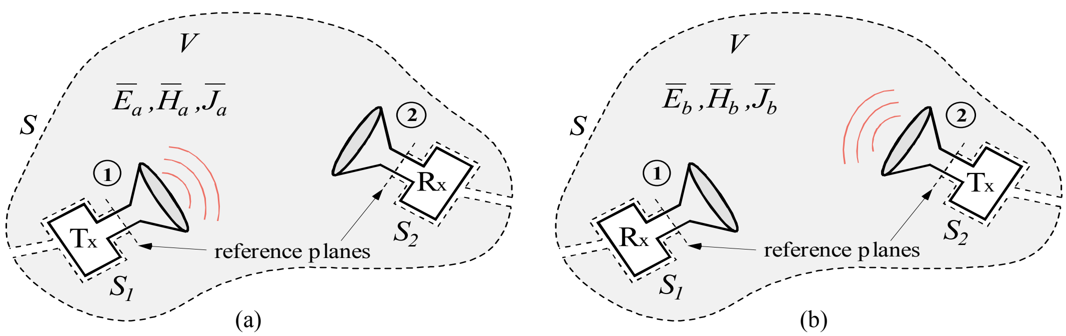


Figure 3.15 Generic point-to-point link. (a) Antenna #1 is the transmitter; (b) antenna #2 is the transmitter.

impressed sources in the problem are located in the transmitter. We can exclude these sources from the volume V by deforming the surface S to include surfaces S_1 and S_2. There can still be currents flowing inside of V (on the antennas or other bodies of matter) but they will all be *induced* currents. The fields and currents inside V in each of the two situations of fig. 3.15 are denoted by $(\overline{E}_a, \overline{H}_a, \overline{J}_a)$ and $(\overline{E}_b, \overline{H}_b, \overline{J}_b)$, respectively. Letting the surface S extend to infinity and applying the Lorentz reciprocity theorem gives

$$\oiint_{S_1+S_2} \left[\overline{E}^a \times \overline{H}^b - \overline{E}^b \times \overline{H}^a\right] \cdot d\overline{S} = \iiint_V \left[\overline{E}^b \cdot \overline{J}^a - \overline{E}^a \cdot \overline{J}^b\right] dV \tag{3.33}$$

Since there are no impressed currents in the volume (by design), the volume itegral vanishes, assuming the medium within the volume is reciprocal. Furthermore, if we assume the transmitter and receiver and feed lines are shielded by a perfect conducting surface, then the surface integral terms will vanish everywhere along S_1 and S_2, *except* over the cross sections of the feed lines at the reference planes. Therefore we have

$$\iint_{\mathrm{RP}_1+\mathrm{RP}_2} \left(\overline{E}^a \times \overline{H}^b - \overline{E}^b \times \overline{H}^a\right) \cdot d\overline{S} = 0 \tag{3.34}$$

where RP_i is the cross-sectional surface defined by the ith reference plane. If the reference planes are chosen sufficiently far from the antenna so that higher-order (evanescent) modes have dies away, then the fields in these integrals are therefore described in terms of the dominant mode fields. Using normalized mode functions we can write the tangential fields in the transmission lines as superpositions of forward and reverse traveling-waves,

$$\overline{E}_T = \left(V^+ e^{-j\beta\xi} + V^- e^{+j\beta\xi}\right) \overline{e}_0 \tag{3.35a}$$

$$\overline{H}_T = \frac{1}{Z_0}\left(V^+ e^{-j\beta\xi} - V^- e^{+j\beta\xi}\right) (\hat{\xi} \times \overline{e}_0) \tag{3.35b}$$

where Z_0 is the characteristic impedance, β is the propagation constant on the feed line, ξ represents the axial direction (direction of wave propagation) on the feed line, and $\overline{e}$ is the normalized transverse mode function for the dominant mode satisfying

$$\iint_{\mathrm{RP}} \overline{e}_0 \cdot \overline{e}_0 \, dS = 1 \tag{3.36}$$

In this form, and taking the reference planes at $\xi = 0$ in each feed line, the total voltage and current at the reference planes can be written as

$$V = V^+ + V^- \tag{3.37a}$$

$$I = \frac{V^+}{Z_0} - \frac{V^-}{Z_0} \tag{3.37b}$$

Substituting the fields (3.35) into (3.34) and using (3.36)-(3.37) gives

$$V_1^a I_1^b + V_2^a I_2^b = V_1^b I_1^a + V_2^b I_2^a \tag{3.38}$$

where the superscript a or b denotes the physical situations of fig. 3.15.

We have reduced the system to a two port network, as shown in fig. 3.16. The voltages and currents at the reference planes can be expressed in terms of the equivalent impedance parameters of the two-port

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \tag{3.39a}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \tag{3.39b}$$

The Z-parameters of this two-antenna system are independent of the transmitter and receiver configuration, depending only on the antennas themselves and their separation and the intervening medium. Using (3.39) in (3.38) gives

$$Z_{12} = Z_{21}$$

which is the familiar statement of reciprocity in circuit theory. Parameters Z_{12} and Z_{21} are called the *transfer impedances* for the system, and are responsible for the coupling between the two

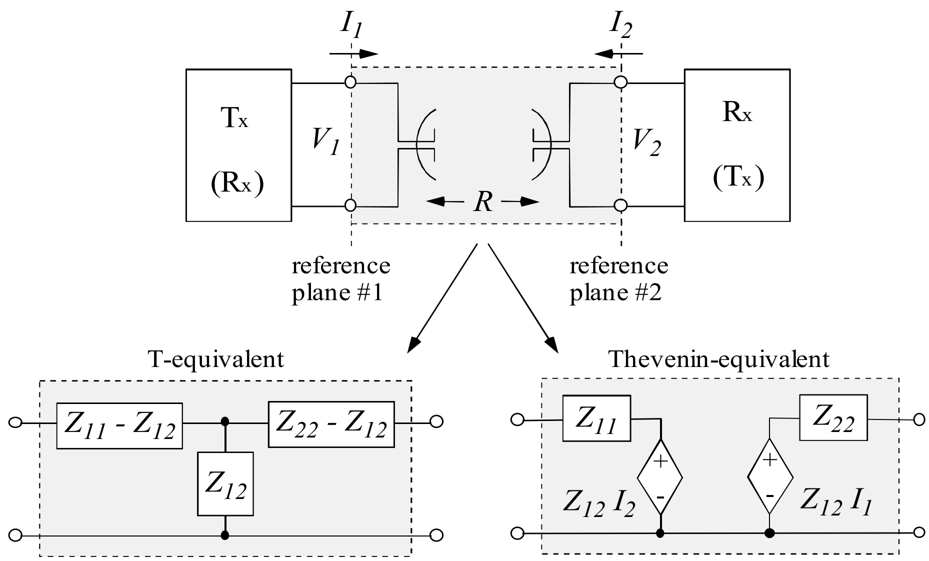


Figure 3.16 Two-port circuit representation of the generic communications link of fig. 3.15.

antennas. The Thevenin equivalent for the two-port, shown in fig. 3.16, gives a nice illustration of the role played by Z_{12}.

Clearly the transfer impedances are a function of R, the distance between the two antennas. In the limit of infinite separation, we would expect no coupling, hence

$$\lim_{R\to\infty} Z_{12} = 0$$

In this limit the driving point impedance of each antenna is then

$$\lim_{R\to\infty} \frac{V_1}{I_1} \approx Z_{11} \equiv Z_{a1} \qquad \lim_{R\to\infty} \frac{V_2}{I_2} \approx Z_{22} \equiv Z_{a2}$$

where Z_{a1} and Z_{a2} are the antenna impedances measured in isolation.

Consider the link when antenna #1 is transmitting, and antenna #2 is terminated in a receiver with an impedance Z_L, as in fig. 3.17. In this case $V_2 = -I_2 Z_L$, so from (3.39) we find

$$I_2 = -\frac{Z_{12}}{Z_{22} + Z_L} I_1$$

and hence

$$V_1 = I_1 \left[Z_{11} - \frac{Z_{12}^2}{Z_{22} + Z_L} \right] \tag{3.40}$$

The voltage at terminal #1 has two contributions, one from the driving current I_1, and the other an induced voltage due to backscatter from antenna #2, sometimes referred to as the contribution from "mutual coupling". From the perspective of the transmitter, the link can be therefore be replaced by an equivalent impedance Z_{in} given by

$$Z_{in} = Z_{a1} - \frac{Z_{12}^2}{Z_{a2} + Z_L} \tag{3.41}$$

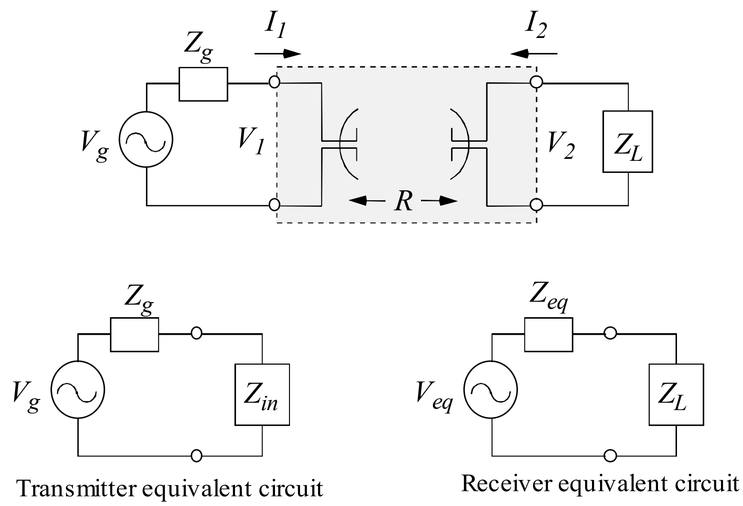


Figure 3.17 Equivalent circuits for a typical antenna link.

When the antennas are close to each other, the mutual coupling term can contribute significantly to the impedance. On the other hand, when the antennas are in the far-field of each other, then we expect Z_{12} to be small, and then

$$Z_{in} \approx Z_{a1} \qquad \text{(far-field)} \tag{3.42}$$

So as long as the antennas are far apart, they are essentially uncoupled as far as the transmitter is concerned, and only the intrinsic impedance Z_{a1} of antenna #1 is necessary for design of the impedance matching networks.

Similarly, from the perspective of the receiver we can replace the link by a Thevenin equivalent circuit (fig. 3.17), where V_{eq} and Z_{eq} can be easily found as

$$V_{eq} = Z_{12}I_1 = V_g \frac{Z_{12}}{Z_g + Z_{in}} \qquad\qquad Z_{eq} = Z_{a2} - \frac{Z_{12}^2}{Z_{a1} + Z_g} \tag{3.43}$$

When the antennas are far apart, the receiver equivalent circuit parameters become

$$V_{eq} = V_g \frac{Z_{12}}{Z_g + Z_{a1}} \qquad\qquad Z_{eq} = Z_{a2} \qquad \text{far-field)} \tag{3.44}$$

Notice that in the far-field case the open-circuit voltage V_{eq} does not depend on the load Z_L, whereas in the general case (3.43) it does, indirectly, through changes in Z_{in} due to mutual coupling. So, in a receiving mode in the far-field, an antenna behaves like a source with an intrinsic impedance equal to the driving point impedance when used as a transmitter. One must be careful about interpreting this equivalent circuit physically—it is tempting to regard power absorbed in the equivalent generator impedance Z_{eq} as the total scattered power, but this is not true in general (see [**?**], section 4.9). The equivalent circuit is physically meaningful only insofar as determining the power delivered to the load (receiver) impedance.

We can now prove that the receiving and transmitting properties of an antenna are the same. Consider the situation shown in fig. 3.18. We apply a fixed current to antenna #1 and move

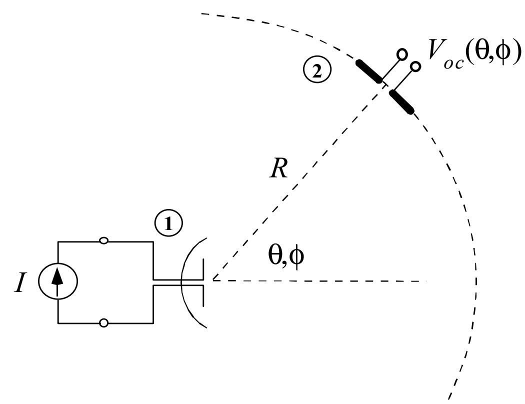


Figure 3.18 Arrangement for demonstrating equivalence of transmitting and receiving properties, as discussed in the text.

antenna #2 along a constant radius circle in the far-field of #1. As long as antenna #2 is always oriented for a polarization match then the measured open-circuit voltage records the transmitting pattern of antenna 1; *i.e.* $Z_{21} = V_{oc}(\theta, \phi)/I$ is the normalized field pattern. If we reverse the arrangement so that the current generator feeds antenna #2 and antenna #1 is recieving, then Z_{12} is the normalized receiving pattern. By reciprocity, $Z_{21} = Z_{12}$, and hence we have the desired result: *for all antenna systems obeying reciprocity, the transmitting pattern and receiving pattern are equivalent.*

3.2.2 Receiving Properties: Effective Length and Effective Area

We have reduced the point-to-point transmission link to an equivalent circuit without discussing how to calculate the equivalent circuit parameters. In fact, calculating these parameters is a large part of antenna theory. However, there are alternative ways to describe energy transfer between antennas. Important link calculations can be computed using the concepts of effective receiving area and effective length.

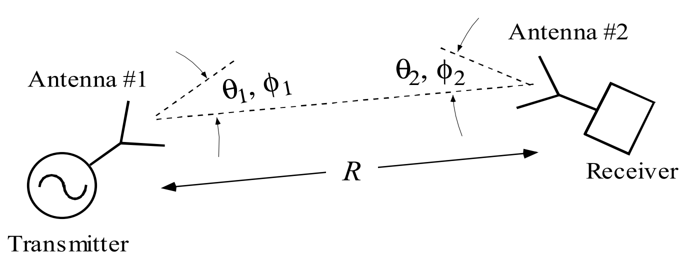


Figure 3.19 Point-to-point antenna link.

In the system shown in figure 3.19, the power density indent on receiving antenna is

$$P_{\text{inc}} = \frac{P_{\text{in}}}{4\pi R^2} G_1(\theta_1, \phi_1). \tag{3.45}$$

The receiving antenna will capture some fraction of this incident field and can deliver a power P_{rec} to a matched load. We relate P_{rec} to P_{inc} by defining an *effective area* A_{eff} so that

$$P_{\text{rec}} = P_{\text{inc}} A_{\text{eff}}(\theta_2, \phi_2) \tag{3.46}$$

The effective area is sometimes referred to as the *capture cross section* or *absorbing cross-section*. Now if the transmitter and receiver are interchanged, then by reciprocity the power received should be the same, so

$$\frac{P_{\text{in}}}{4\pi R^2} G_1(\theta_1, \phi_1) A_{\text{eff}_2}(\theta_2, \phi_2) = \frac{P_{\text{in}}}{4\pi R^2} G_2(\theta_2, \phi_2) A_{\text{eff}_1}(\theta_1, \phi_1) \tag{3.47}$$

and hence

$$\frac{G_1(\theta_1, \phi_1)}{A_{\text{eff}_1}(\theta_1, \phi_1)} = \frac{G_2(\theta_2, \phi_2)}{A_{\text{eff}_2}(\theta_2, \phi_2)} \tag{3.48}$$

Since we have made no assumptions about the antennas (other than constraints associated with the reciprocity theorem) then the quantity A_{eff}/G must be a universal constant independent of the antenna and the receiving direction. We will also prove this using reciprocity in the next section, and again in Chapter 4 using statistical mechanics.

Effective Area of an Antenna

We have alread shown how the reciprocity theorem can link the transmitting and receiving properties of an antenna, so it is natural to revisit the reciprocity theorem to relate the antenna gain to the effective area. To do this we use a physical situation like that of fig. 3.15, but with antenna #2 replaced by a Hertzian dipole, a simple radiator whose fields we have previously computed. This situation in shown in fig. 3.20. In fig. 3.20a, the generic antenna is radiating and producing the fields $(\overline{E}_a, \overline{H}_a)$, and in fig. 3.20b it is receiving in the presence of he impressed current $\overline{J}_b$ which produces the fields $(\overline{E}_b, \overline{H}_b)$.

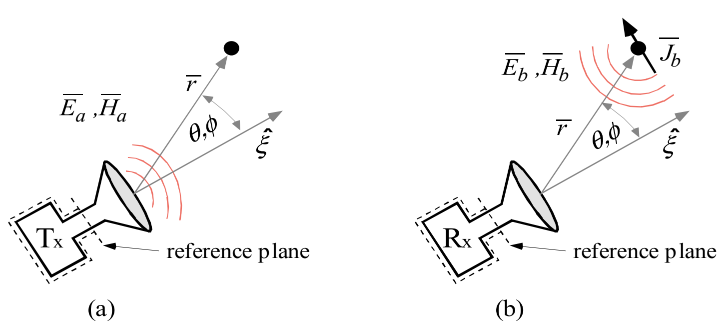


Figure 3.20 (a) Transmitting antenna producing the fields $(\overline{E}_a, \overline{H}_a)$. (b) Same antenna used to receive the fields $(\overline{E}_b, \overline{H}_b)$ generated by a testing dipole $\overline{J}_b$.

Also show equivalent circuits.

We assume that the antenna is matched to the transmitter and receiver such that $Z_g = Z_a^*$. Writing $Z_a = R_a + \jmath X_a$ and $Z_g = R_g + \jmath X_g$, this implies that$R_g = R_a$ and $X_g = -X_a$.

The reciprocity theorem gives

$$\iint_{RP} \left[\overline{E}_a \times \overline{H}_b - \overline{E}_b \times \overline{H}_a\right] \cdot d\overline{S}' = -\iiint_V \overline{J}_b \cdot \overline{E}_a \, dV' \tag{3.49}$$

We have already evaluated the left-hand side in (). Writing the impressed curret as $\overline{J}_b = (I_0 d\ell)\hat{\zeta}\delta(\overline{r})$ gives

$$-V_1^a I_1^b + V_1^b I_1^a = -(I_0 d\ell)\hat{\zeta} \cdot \overline{E}_a$$

Using the equivalent circuits in fig. 3.20 we can write

$$I_1^a I_1^b \underbrace{(Z_a + Z_g)}_{2R_a} = (\hat{\zeta} \cdot \hat{E}_a) E_a I_0 d\ell$$

Formign the magnitude squared of both sides, we can write

$$|I_1^a|^2 |I_1^b|^2 (2R_a)^2 = \underbrace{|\hat{\zeta} \cdot \hat{E}_a|^2}_{\text{PLF}} |E_a|^2 (I_0 d\ell)^2 \tag{3.50}$$

where PLF is the polarization loss factor, to be defined later. Note that ζ describes the direction of $\overline{J}_b$ and hence the polarization of the incident wave. We can orient this vector anyway we wish; taking $\hat{\zeta} = \hat{E}_a^*$ makes PLF=1 in general. Now, on the right-hand side, we can relate the field E_a to the gain of the antenna as

$$\frac{|E_a|^2}{2\eta} = \frac{P_t}{4\pi r^2} G(\theta, \phi) = \frac{1}{2} |I_1^a|^2 R_a \frac{G(\theta, \phi)}{4\pi r^2} \tag{3.51}$$

In addition, we can relate the current I_1^b to the effective area as

$$|I_1^b|^2 = \frac{2\mathcal{P}_{inc} A_{eff}}{R_a} = \frac{2A_{eff}}{R_a} \frac{|E_b|^2}{2\eta}$$

The field produced by the Hertizian dipole is

$$\overline{E}_b = \jmath\omega\mu(I_0 d\ell) \frac{e^{-\jmath kr}}{4\pi r} \hat{\zeta}$$

so we have

$$|I_1^b|^2 = \frac{A_{eff}\eta}{R_a 4\lambda^2 r^2} (I_0 d\ell)^2 \tag{3.52}$$

Sustituting () and () into () and collecting terms gives the result

$$A_{eff} = \frac{\lambda^2}{4\pi} G(\theta, \phi) \tag{3.53}$$

which is the desired result.

Effective Length of an Antenna

Just as the effective area is defined to give the correct received power given a certain incident power density, we can also define an *effective length* of an antenna to relate the induced voltage or current at the terminals to the incident field intensity through

$$V_{oc} = \bar{\ell}_{\text{eff}} \cdot \overline{E}_{\text{inc}} \tag{3.54}$$

The effective length also describes the polarization characteristics of the antenna.

We can use the reciprocity theorem to quantify the effective length. In fig. **??** is the antenna in question, used in both the transmit and receive mode. In the first case a transmitter is connected to the antenna, producing a field $\overline{E}_a$ at the observation point, $\overline{r}$. In the second case a Hertzian dipole is placed at $\overline{r}$ and the antenna is configured to receive the radiation. From the reciprocity theorem we have, using the same principles as before,

$$\iint_{RP} \left[\overline{E}_a \times \overline{H}_b - \overline{E}_b \times \overline{H}_a\right] \cdot d\overline{S}' = -\iiint_V \overline{J}_b \cdot \overline{E}_a \, dV' \tag{3.55}$$

where the surface RP is the cross section of the feeding structure at a specified reference plane, and V is the volume outside of S. We would like to derive an expression for the open-circuit voltage induced at the terminals in terms of an incident field. Therefore in Case-b the receiver is configured so that there is an open circuit condition at RP, hence $\overline{H}_b = 0$ and $\overline{E}_b = V_{oc}\bar{e}_0$, where $\bar{e}_0$ is the dominant mode field function. In Case-a, the fields on the feed line can similarly be written in terms of mode function so that

$$\overline{E}_a = V_0 \bar{e}_0 \qquad\qquad \overline{H}_a = I_0(\hat{\xi} \times \bar{e}_0) \tag{3.56}$$

where V_0 and I_0 are the terminal voltage and current in the transmit mode, and ξ is the axial direction on the feed line. The surface integral then evaluates to

$$-V_{oc}I_0 \iint_{RP} \left[\bar{e}_0 \times (\hat{\xi} \times \bar{e}_0)\right] \cdot \hat{\xi} \, dS = -V_{oc}I_0 \tag{3.57}$$

For any antenna, we know that when the observation point is in the far field then, we can write the fields in the form

$$\overline{E}_a(r, \theta, \phi) = \frac{\jmath\omega\mu}{4\pi} \frac{e^{-jkr}}{r} \overline{f}(\theta, \phi) \tag{3.58}$$

where $\overline{f}$ is the pattern function. This equation defines f. For the Hertzian "testing" dipole in Case-b,

$$\overline{J}_b = (Id\bar{\ell})\delta(\overline{r}) \tag{3.59}$$

we have

$$V_{oc}I_0 = \underbrace{\frac{\jmath\omega\mu}{4\pi} e^{-jkr} Id\bar{\ell}}_{\overline{E}_{\text{inc}}} \cdot \overline{f}(\theta, \phi) \tag{3.60}$$

where we have identified $\overline{E}_{\text{inc}}$ as the field incident on the antenna from the testing dipole. Therefore the effective length of the antenna is just

$$\bar{\ell}_{\text{eff}} = \frac{1}{I_0} \overline{f}(\theta, \phi) \tag{3.61}$$

where I_0 is the current at the *antenna terminals*. Notice that the effective length allows us to write the field *produced* by an antenna in the form

$$\overline{E} = \frac{\jmath\omega\mu}{4\pi} (I_0 \bar{\ell}_{\text{eff}}) e^{-jkr} \tag{3.62}$$

3.2.3 Short-Circuit Current and Open-Circuit Voltage

We have just shown how to calculate the open-circuit voltage in a receiving antenna using reciprocity concepts. The following is alternate derivation (see Chapter 13 of [**?**]) that expands on this result and gives us another chance to practice applying the reciprocity theorem to antenna problems. We will start by deriving an expression for the short-circuit current in a receiving antenna.

We start with the arrangement shown in fig. 3.21, which is physically similar to fig. 3.20. In fig. 3.21a, a PEC transmitting antenna driven by a voltage generator V (modelled as a magnetic current loop $\overline{M}$) generates total fields $(\overline{E}, \overline{H})$ and a feed current I. In fig. 3.21b an impressed

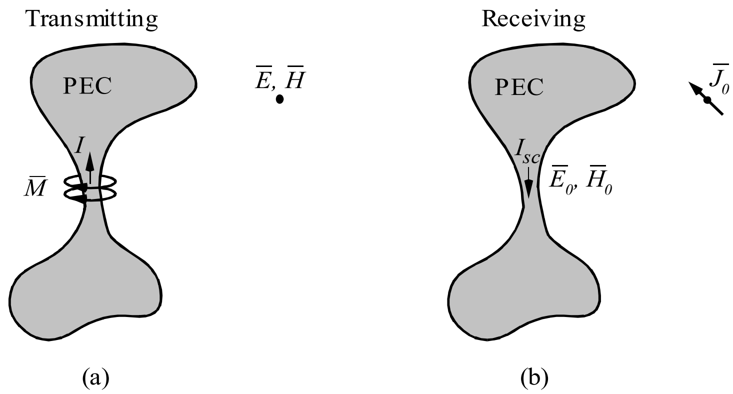


Figure 3.21 (a) Transmitting antenna producing the fields $(\overline{E}, \overline{H})$. (b) Same antenna used to receive the fields $(\overline{E}_0, \overline{H}_0)$ generated by a testing dipole $\overline{J}_0$.

current $\overline{J}_0$ generates the total fields $(\overline{E}_0, \overline{H}_0)$ in the presence of the same antenna when configured as a receiver, in this case under short circuit conditions. This time we enclose the problem by a surface at infinity, so the reciprocity theorem () becomes

$$\iiint \left[\overline{E}_0 \cdot \overline{J} - \overline{H}_0 \cdot \overline{M}\right] \, dV = \iiint \overline{E} \cdot \overline{J}_0 \, dV \tag{3.63}$$

The first term on the left, $\overline{E}_0 \cdot \overline{J}$ is zero since the tangential electric field must vanish on the surface of the conductor. The second term can be evaluated as we discussed in Chapter 1 in connection with sources

$$-\iiint \overline{H}_0 \cdot \overline{M} \, dV = -V \oint_{loop} \overline{H}_0 \cdot d\overline{\ell} = V I_{sc}$$

so that

$$I_{sc} = \frac{1}{V} \iiint \overline{E} \cdot \overline{J}_0 \, dV \tag{3.64}$$

Now, in section 2.x we found that when the transmitting antenna is replaced by a Love equivalent (currents flowing in free space), the reciprocity theorem relates the fields to those of a testing dipole $\overline{J}_i$ as

$$\iiint \overline{E} \cdot \overline{J}_i \, dV = \iiint \overline{E}_i \cdot \overline{J} \, dV \tag{3.65}$$

In this case, the field E_i is the field produced by the testing dipole $\overline{J}_i$ in free space with no other objects around. Therefore, if we make $\overline{J}_0 = \overline{J}_i$ we can also write (3.64) as

$$I_{sc} = \frac{1}{V} \iiint \overline{E} \cdot \overline{J}_i \, dV = \frac{1}{V} \iiint \overline{E}_i \cdot \overline{J} \, dV \tag{3.66}$$

3.2.4 Reception of Completely Polarized Waves

The polarization characteristics of an antenna are usually specified in term of the transmitted wave, described by the pattern function $\overline{f}$. In other words, the antenna polarization is described in terms of the direction pointing *away* from the antenna, whereas the polarization of a received signal is usually specified in terms of the direction of propagation, which is *towards* the antenna. This often leads to confusion in the case of circular- or elliptically-polarized waves. If the antenna is designed to transmit CW polarization in a given direction, it will receive CCW polarized signals incident from that direction. If the polariation of the incident beam is descrbed by the vector ξ, then the received signal will be proportional to

$$\text{PLF} = \frac{|\overline{f}^* \cdot \xi|^2}{|\overline{f}|^2 \, |\xi|^2} = |\hat{f}^* \cdot \hat{\xi}|^2 \tag{3.67}$$

This is called the *polarization loss factor*, and describes the fraction of incident power that has the correct polarization for reception. The complex conjugate of $\overline{f}$ corrects for the opposite sense of polarization when receiving. If $\hat{f}^* \cdot \overline{\xi} = 1$, the antenna is said to be *polarization-matched* to the incoming signal, and PLF=1. However, if the incoming signal is orthogonal to the antenna polarization such that $\hat{f}^* \cdot \overline{\xi} = 0$, then no signal will be recieved. For example, suppose the antenna is designed to transmit CW polarization in the $\hat{z}$ direction so that $\overline{f} = \hat{x} - \jmath\hat{y}$. The polarization loss factor for various received signal polarizations is

$$\begin{aligned}
\xi &= \hat{x} && \text{Linear} && \Rightarrow \quad PLF = \frac{1}{2} \\
\xi &= \hat{x} + \jmath\hat{y} && \text{CW} && \Rightarrow \quad PLF = 0 \\
\xi &= \hat{x} - \jmath\hat{y} && \text{CCW} && \Rightarrow \quad PLF = 1
\end{aligned}$$

So an antenna that transmits CW radiation receives CCW radiation, and vice versa.

Give example of polarization interleaving in DBS downlinks.

We have tacitly assumed that the signal to be received in completely polarized, meaning that it is a coherent signal in one definite and perpetual polarization. In contrast, incoherent electromagnetic emission from natural objects such as celestial bodies) arrives at an antenna with a randomly changing polarization state. If the polarization state varies lowly relative to centroid of frequency, this is called a *partially polarized* wave. This is of importance in the field of radio astronomy and also passive radiometric detection for remote sensing applications. Howeve it is a relatively specialized topic that will not be considered in this work; the interested reader is referred to Chapter 3 of [**?**].

3.2.5 Equivalent Circuit Parameters

Now that we have l_{eff} and A_{eff}, we can find Z21 an Z12!

3.3 METHODS FOR FINDING THE TERMINAL IMPEDANCE

As we have seen, the terminal impedance is an important parameter for characterizing the antenna. There are some general methods for calculating this impedance which we now consider. These methods are often referred to collectively as "Induced EMF" methods, although the concept of an induced EMF (electro-motive force) does not always play an explicit role in the derivation. This terminology has its roots in the physical origin of the radiation resistance.

3.3.1 Induced EMF method

The antenna can be represented by a complex terminating impedance (or admittance), where the real part of the impedance is associated with an average power flow away from the antenna (radiation), and the reactive part is associated with the non-radiative "near" fields of the antenna. The field description can be given a mechanical interpretation by considering the motion of electrons on the antenna. When the electrons are accelerated by an applied field (from the generator), they will radiate and hence lose energy. This change in energy is equivalent to a force acting against the motion of the electrons. This force, called the "induced EMF" or "radiation reaction", is attributed to the radiated fields acting back on the current. Mechanically it can be thought of as the recoil force on the electrons as they eject photons. In order to maintain a certain time-varying current distribution on the antenna, the generator must therefore expend energy to overcome the radiation reaction.

This viewpoint allows us to formulate a simple expression for the input impedance of the antenna. For simplicity we first consider a simple dipole, as shown in fig. 3.22; the concept will be generalized in the next section. An applied voltage V_{in} at the terminals of this antenna will induce a current flow given by the distribution $I(z)\hat{z}$. This current in turn produces a "scattered" field $\overline{E}_{\text{scatt}}$. Note that the scattered field at any point along the antenna includes contributions

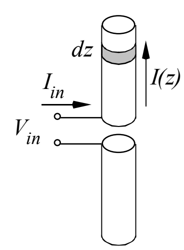


Figure 3.22 Simple dipole antenna.

from the induced currents on all parts of the antenna, not just the currents at the point in question. The induced current must distribute itself such that the induced field plus the applied field satisfy the boundary conditions on the antenna. In circuit terms, the scattered field is an "induced EMF", distributed along the antenna. The induced EMF over a length dz is therefore

$$dV(z) = -\overline{E}_{\text{scatt}}(z) \cdot \hat{z}\, dz$$

This EMF can be related to a corresponding change in terminal current using reciprocity. Since an applied voltage V_{in} at the terminals produces a current $I(z)$ at the point z, then by reciprocity an EMF dV at point z will produce a current dI_{in} at the terminals

$$\frac{V_{in}}{I(z)} = \frac{dV(z)}{dI_{\text{in}}}$$

which gives

$$V_{\rm in} dI_{\rm in} = -\overline{E}_{\rm scatt}(z) \cdot \overline{I}(z)\, dz$$

Integrating this expression over the length of the antenna gives

$$V_{\rm in} I_{\rm in} = -\int \overline{E}_{\rm scatt}(z) \cdot \overline{I}(z)\, dz \tag{3.68}$$

The input impedance is then

$$Z_{\rm in} = \frac{V_{\rm in}}{I_{\rm in}} = -\frac{1}{I_{\rm in}^2} \int \overline{E}_{\rm scatt}(z) \cdot \overline{I}(z)\, dz \tag{3.69}$$

This is the so-called *induced EMF formula* for the input impedance of a dipole. If the true currents and fields for the structure are used, then this expression is exact insofar as the ideal dipole structure is concerned. *However, if we already knew the true current distribution, there would be no reason to use (3.69), since we would just compute the impedance from $Z_{\rm in} = V_{\rm in}/I_{\rm in} = V_{\rm in}/I(0)$!!* The induced EMF result is therefore only used in situations where the current distribution can only be guessed at, or otherwise approximated. It is attractive in this respect due to a stationary property which will be discussed in the next section.

Once we have a trial current distribution for the antenna, (3.69) requires that we compute the fields produced by those currents. If we assume that the applied field is concentrated in the feed gap region, the applied field can be taken as $\overline{E}_{inc} = \hat{z} V_{\rm in}/g$ in the gap, where g is the gap width, and zero elsewhere along the conductors (see discussion of sources in Chapter 1). This applied field induces whatever current is necessary to maintain a voltage $V_{\rm in}$, or equivalently a field $-\hat{z} V_{\rm in}/g$, in the gap, and a vanishing total field along the conductors. Therefore the true current $I(z)$ will produce a scattered field which is

$$\overline{E}_{\rm scatt} = \begin{cases} -\hat{z} V_{\rm in}/g & \text{in the gap} \\ 0 & \text{along the conductor} \end{cases}$$

When this is inserted in (3.69), we can see that the integration is just over the gap region. However, it should be noted that the fields produced by an approximation to the true currents will not necessarily vanish along the conductors, and hence the integration must then be carried out over the entire length of the antenna. In fact, it has been found that relatively small changes in the current distribution can lead to enormous changes in the tangential fields. This makes the stationary character of (3.69) even more remarkable.

3.3.2 Generalized Impedance From Reciprocity and its Variational Property

The derivation above, based on circuit principles, is limited to thin wire antennas where the current is filamentary. A more general expression can be derived from a field statement of reciprocity as follows. We start with an arbitrary antenna as shown in fig. 3.23a, which is fed by a voltage generator. Using field-equivalence concepts we can replace the conductors and generators by free space, as shown in fig. 3.23b. The impressed surface currents $\overline{M}_a$ and J_a in this case can be related to the applied field and induced surface currents, respectively, from the original problem. The combination of these currents produces the same fields *outside* of S as exist in the original problem, and produce a null field within V. We now introduce an auxiliary problem in fig. 3.23c. This concerns exactly the same volume as in fig. 3.23b, but introduces a different current J_b

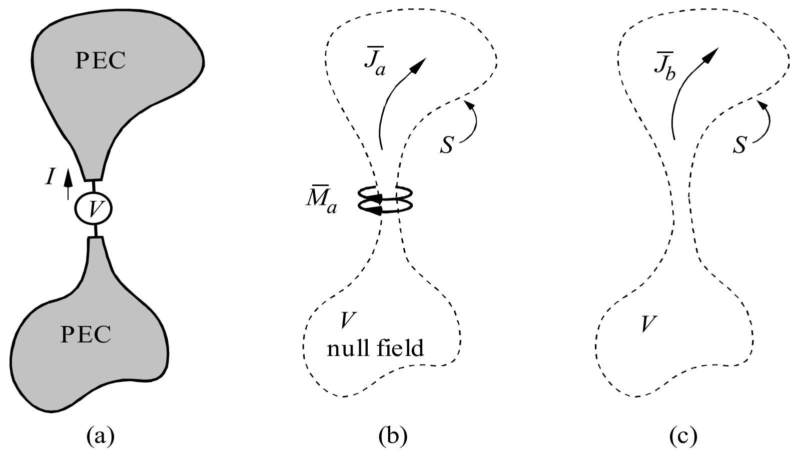


Figure 3.23 (a) A PEC antenna with feed. (b) Using equivalence principles, the feed is replaced by an impressed magnetic current around the surface S, and the conductors are replaced by free-space. (c) An auxiliary current distribution flowing in the same volume V.

flowing in free-space within the volume. The reciprocity theorem links the currents and fields of the two cases according to

$$\iiint_V \overline{E}_a \cdot \overline{J}_b \, dV = \iiint_V \overline{E}_b \cdot \overline{J}_a \, dV - \iiint_V \overline{H}_b \cdot \overline{M}_a \, dV \tag{3.70}$$

where ($\overline{E}_a$,$\overline{H}_a$) are the total fields produced by the currents ($\overline{J}_a$,$\overline{M}_a$). The left hand side of (3.70) is therefore zero since the tangential electric field is zero along S in fig. 3.23b (the tangential electric field is non-zero just outside of the current distribution $\overline{M}_a$, given by the negative of the applied field, but it is zero on and within S). The second integral on the right of (3.70) can be evaluated as in (**??**) to give

$$-V_a I_b = \iiint_V \overline{E}_b \cdot \overline{J}_a \, dV$$

where V_a is the original generator voltage, and I_b is the total current passing through the feed region in fig. 3.23c. Therefore the driving point impedance is given by

$$Z_{in} \equiv \frac{V_a}{I_a} = -\frac{1}{I_a I_b} \iiint_V \overline{E}_b \cdot \overline{J}_a \, dV \tag{3.71}$$

This is a remarkable result, since the auxiliary current $\overline{J}_b$ can be specified arbitrarily. In some cases it may be advantageous to use a simple current distribution in order to simplify the computation of impedance. However, the formula is only exact when the true currents $\overline{J}_a$ are used, and as we saw in the previous section, if the true currents were already known there would be no reason to use this result. It is mostly used for situations where the current is approximated, and in those cases it is advantageous to use the specific choice of $\overline{J}_b = \overline{J}_a \equiv \overline{J}$ (from which follows $I_b = I_a \equiv I$), so that

$$Z_{in} = -\frac{1}{I^2} \iiint_V \overline{E} \cdot \overline{J} \, dV \tag{3.72}$$

where $\overline{E}$ is now the field produced by the current J. This is the desired generalization of the induced EMF result for the input impedance of an antenna. Note that this result is essentially identical with (**??**) for the impedance parameters of a multiport network developed in Chapter 1.

The utility of this expression derives from the fact that it is stationary with respect to small errors in the assumed current. If the true currents and fields are $\overline{J}_0$ and $\overline{E}_0$, respectively, then a small error in the assumed current can be represented as $\overline{J} = \overline{J}_0 + \delta\overline{J}$, and all other derived quantities will be similarly perturbed,

$$\overline{E} \to \overline{E}_0 + \delta\overline{E} \qquad I \to I_0 + \delta I \qquad Z_{in} \to Z_{in} + \delta Z_{in}$$

Substituting these perturbed quantities into (3.72) gives

$$Z_{in} + \delta Z_{in} = -\frac{1}{I_0^2}\left(1 - \frac{2\,\delta I}{I_0}\right)\iiint \left[\overline{E}_0 \cdot \overline{J}_0 + \delta\overline{E} \cdot \overline{J}_0 + \overline{E}_0 \cdot \delta\overline{J} + \delta\overline{E} \cdot \delta\overline{J}\right] dV$$

From reciprocity we find $\delta\overline{E} \cdot \overline{J}_0 = \overline{E}_0 \cdot \delta\overline{J}$; keeping only first order terms gives

$$\delta Z_{in} = \frac{1}{I_0^2}\left[\frac{2\,\delta I}{I_0}\underbrace{\iiint \overline{E}_0 \cdot \overline{J}_0\,dV}_{-V_0 I_0} - 2\underbrace{\iiint \overline{E}_0 \cdot \delta\overline{J}\,dV}_{-V_0\,\delta I}\right] = 0$$

which shows that *the impedance is unchanged, or stationary, with respect to small perturbations in the current*. Expressions which have this property can often be constructed for physically meaningful parameters, and are called *variational* (from variational calculus). A guess current that has some error $\delta\overline{J}$ will give an impedance accurate to the order of $(\delta\overline{J})^2$. Note that the result (3.71) does not possess this property in the general case when the two currents $\overline{J}_a$ and $\overline{J}_b$ are different.

In general, we are usually concerned with antennas constructed of perfect conductors as in fig. **??**a, in which case the currents are always surface currents, and the impedance is then given by

$$Z_{\text{in}} = -\frac{1}{I^2}\oiint_S \overline{E}(\overline{r}) \cdot \overline{J}_s(\overline{r})\,dS \tag{3.73}$$

The electric field $\overline{E}$ is that produced by the impressed current distribution $\overline{J}_s$, and can be expressed as

$$\overline{E}(\overline{r}) = -\jmath\omega\mu \oiint_S \overline{J}_s(\overline{r}') \cdot \overline{\overline{G}}(\overline{r}, \overline{r}')\,dS' \tag{3.74}$$

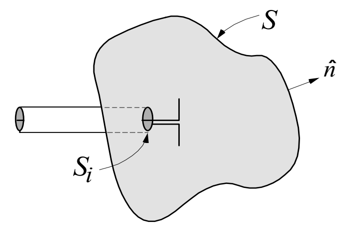
Using equivalence principles, the conductors can usually be replaced by free-space so that the unbounded Green's function can be used. The self-impedance is then given by

$$Z_{\text{in}} = -\frac{\jmath\omega\mu}{I^2}\oiint_S \oiint_S \overline{J}_s(\overline{r}') \cdot \overline{\overline{G}}_0 \cdot \overline{J}_s(\overline{r})\,dS'\,dS \tag{3.75}$$

3.3.3 Power conservation methods

Poynting's theorem and power conservation arguments also provide a useful technique for finding the antenna impedance. In fact, we already used this approach to find the radiation resistance of the Hertzian dipoles. Consider the situation of figure 3.24, where a transmission-line delivers energy to some arbitrary element Z. The element is surrounded by a closed surface, S, with only a

Figure 3.24 Illustration for finding the antenna impedance at reference plane S_i via Poynting's theorem and power conservation arguments.



small aperture S_i through which energy is coupled to/from the transmission line. At this aperture plane there exists a voltage V_i and current I_i, which are related to the fields by

$$\frac{1}{2} V_i I_i^* = -\frac{1}{2} \iint_{S_i} \left(\overline{E} \times \overline{H}^* \right) \cdot \hat{n} \, dS$$

where $\hat{n}$ is the unit normal to the surface, directed out of the volume. Using (**??**) and (**??**) allows us to define an impedance at the reference plane S_i as

$$\begin{aligned} Z_{\text{in}} = & \frac{1}{|I|^2} \left[\iiint \sigma |\overline{E}|^2 \, dV \right. \\ & \left. + \; 4 \jmath \omega \iiint \left(\overline{H}^* \cdot \overline{B} - \overline{E} \cdot \overline{D}^* \right) dV + \iint_{S - S_i} \left(\overline{E} \times \overline{H}^* \right) \cdot \hat{n} \, dS \right] \end{aligned} \qquad (3.76)$$

where we have substituted $\overline{J} = \sigma \overline{E}$ for the induced currents in the volume. The last term accounts for complex power flowing out of the volume through the enclosing surface $S - S_i$. If the medium surrounding the antenna in the volume is lossless, then the first volume integral in (3.76) is just the ohmic loss on the antenna. The second integral gives a reactive contribution to the input impedance that is primarily associated with stored energy in the vicinity of the antenna (near fields). The last integral accounts for complex power radiated out of the surface and in general has both a resistive and reactive component.

If the surface S extends to infinity, as in fig. 3.25a, then it can be shown (using the Sommerfeld radiation conditions) that the surface integral term is purely real, and constitutes the *radiation resistance* of the antenna. Then (3.76) reduces to

$$Z_{\text{in}} = R_{\text{loss}} + R_{\text{rad}} + \jmath X(\omega) \qquad (3.77)$$

where

$$R_{\text{loss}} = \frac{1}{|I_i|^2} \iiint \sigma |\overline{E}|^2 \, dV \qquad R_{\text{rad}} = \frac{1}{|I_i|^2} \oiint_S \frac{1}{\eta} |\overline{E}|^2 \, dS$$

$$X(\omega) = \frac{\omega}{|I_i|^2} \text{Re} \iiint \left(\mu |\overline{H}|^2 - \epsilon |\overline{E}|^2 \right) dV$$

Using (3.6) we can write the radiation resistance in terms of the pattern function as

$$R_{\text{rad}} = \frac{1}{\eta |I_i|^2} \oiint_S |f(\theta, \phi)|^2 \, d\Omega \qquad (3.78)$$

The fields and hence the pattern function $\overline{f}(\theta, \phi)$ will be proportional to the feed current I_i, so this term will cancel in the denominator.

A different expression for the impedance can be obtained when the surface enclosing the antenna coincides with the surface of the conductors, as in fig. 3.25b. For simplicity we also

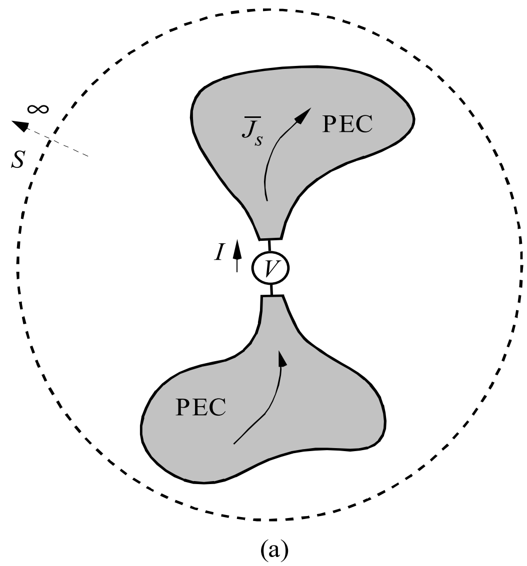


Figure 3.25 (a) PEC antenna enclosed by a spherical surface at infinity. (b) PEC antenna enclosed by a surface S which coincides with the antenna surface, and also encloses the generator.

enclose the generator by the surface as shown. When the conductors are perfectly conducting, the complex power flowing through the surface must equal the power supplied by the generator, so again we have a power conservation statement

$$\frac{1}{2}VI^* = -\frac{1}{2}\oiint_S \left(\overline{E} \times \overline{H}^*\right) \cdot d\overline{S}$$

from which we can find the input impedance as

$$Z_{\text{in}} = \frac{V}{I} = -\frac{1}{|I|^2}\oiint \overline{E} \cdot \overline{J}_s^*\, dS \tag{3.79}$$

where $J_s = \hat{n} \times \overline{H}$ is the equivalent impressed current flowing on the surface S, which includes the induced surface current flowing on the conductors, and a contribution from the feed region (numerically proportional to the current flowing in the generator for small gaps). This is quite similar to the induced EMF formula, except for the complex conjugate on the current density. For real $\overline{J}_s$, the two are identical.

In this formulation, $\overline{E}$ is the total field on the antenna surface, which of course must vanish on the PEC surfaces, so (3.79) implies that

$$Z_{\text{in}} = -\frac{1}{|I|^2}\iint_{\text{feed}} \overline{E} \cdot \overline{J}_s^*\, dS$$

if the true fields and currents are used. This just reinforces the physical expectation that the energy flows out of the gap region, and is merely guided by the conductors into space. This point is worth emphasizing: the antenna structure is simply a device for *guiding* energy away from the generator and into free-space—in fact it is possible to treat the antenna as a finite length of transmission line, where in general the characteristic impedance varies along its length. We will examine this later in connection with the biconical antenna. In many subsequent analysis problems we will focus attention on the induced current distribution on the conductors to compute the radiation patterns and impedances, but it should be remembered that this is just a mathematical device for computing the fields. The most physically correct way to envisage radiation from *any* antenna is to imagine waves emerging and diffracting away from the feed region, and reflecting off the conducting boundaries of the antenna.

3.4 DUALITY AND COMPLEMENTARITY: BOOKER'S RELATION

Any radiation problem involves finding a solution two the two Maxwell curl equations

$$\nabla \times \overline{E} = -\overline{M} - \jmath\omega\overline{B} \tag{3.80a}$$
$$\nabla \times \overline{H} = \overline{J} + \jmath\omega\overline{D} \tag{3.80b}$$

The symmetry of these equations suggests the following *duality principle*: there exists a dual problem which has the same form of solution, but with the roles of $\overline{E}$ and $\overline{H}$ reversed according to the replacements

$$\begin{aligned} \overline{E} &\to \eta\overline{H} \\ \overline{H} &\to -\overline{E}/\eta \\ \overline{J} &\to \overline{M}/\eta \\ \overline{M} &\to -\eta\overline{J} \end{aligned}$$

where $\eta = \sqrt{\mu/\epsilon}$. Using the transformation (3.80), (3.80a) becomes (3.80b) and (3.80b) becomes (3.80a).