Chapter 11

TRANSMISSION LINES

There is a story about four men named Everybody, Somebody, Anybody, and Nobody. There was an important job to be done, and Everybody was asked to do it. Everybody was sure that Somebody would do it. Anybody could have done it, but Nobody did it. Somebody got angry about that, because it was Everybody's job. Everybody thought that Anybody could do it, and Nobody realized that Everybody wouldn't do it. It ended up that Everybody blamed Somebody, when actually Nobody did what Anybody could have done.

—ANONYMOUS

11.1 INTRODUCTION

Our discussion in the previous chapter was essentially on wave propagation in unbounded media, media of infinite extent. Such wave propagation is said to be unguided in that the uniform plane wave exists throughout all space and EM energy associated with the wave spreads over a wide area. Wave propagation in unbounded media is used in radio or TV broadcasting, where the information being transmitted is meant for everyone who may be interested. Such means of wave propagation will not help in a situation like telephone conversation, where the information is received privately by one person.

Another means of transmitting power or information is by guided structures. Guided structures serve to guide (or direct) the propagation of energy from the source to the load. Typical examples of such structures are transmission lines and waveguides. Waveguides are discussed in the next chapter; transmission lines are considered in this chapter.

Transmission lines are commonly used in power distribution (at low frequencies) and in communications (at high frequencies). Various kinds of transmission lines such as the twisted-pair and coaxial cables (thinnet and thicknet) are used in computer networks such as the Ethernet and internet.

A transmission line basically consists of two or more parallel conductors used to connect a source to a load. The source may be a hydroelectric generator, a transmitter, or an oscillator; the load may be a factory, an antenna, or an oscilloscope, respectively. Typical transmission lines include coaxial cable, a two-wire line, a parallel-plate or planar line, a wire above the conducting plane, and a microstrip line. These lines are portrayed in Figure 11.1. Notice that each of these lines consists of two conductors in parallel. Coaxial cables are routinely used in electrical laboratories and in connecting TV sets to TV antennas. Microstrip lines (similar to that in Figure 11.1e) are particularly important in integrated circuits where metallic strips connecting electronic elements are deposited on dielectric substrates.

Transmission line problems are usually solved using EM field theory and electric circuit theory, the two major theories on which electrical engineering is based. In this

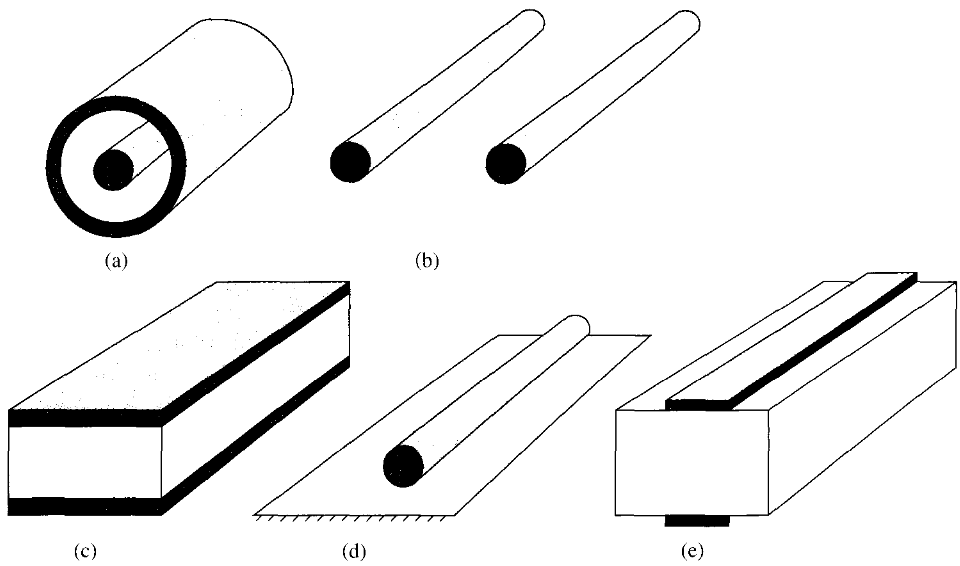


Figure 11.1 Cross-sectional view of typical transmission lines: **(a)** coaxial line, **(b)** two-wire line, **(c)** planar line, **(d)** wire above conducting plane, **(e)** microstrip line.

chapter, we use circuit theory because it is easier to deal with mathematically. The basic concepts of wave propagation (such as propagation constant, reflection coefficient, and standing wave ratio) covered in the previous chapter apply here.

Our analysis of transmission lines will include the derivation of the transmission-line equations and characteristic quantities, the use of the Smith chart, various practical applications of transmission lines, and transients on transmission lines.

11.2 TRANSMISSION LINE PARAMETERS

It is customary and convenient to describe a transmission line in terms of its line parameters, which are its resistance per unit length R, inductance per unit length L, conductance per unit length G, and capacitance per unit length C. Each of the lines shown in Figure 11.1 has specific formulas for finding R, L, G, and C. For coaxial, two-wire, and planar lines, the formulas for calculating the values of R, L, G, and C are provided in Table 11.1 The dimensions of the lines are as shown in Figure 11.2. Some of the formulas[1] in Table 11.1 were derived in Chapters 6 and 8. It should be noted that

1. The line parameters R, L, G, and C are not discrete or lumped but distributed as shown in Figure 11.3. By this we mean that the parameters are uniformly distributed along the entire length of the line.

[1]Similar formulas for other transmission lines can be obtained from engineering handbooks or data books—e.g., M. A. R. Guston, *Microwave Transmission-line Impedance Data.* London: Van Nostrand Reinhold, 1972.

TABLE 11.1 Distributed Line Parameters at High Frequencies*

Parameters	Coaxial Line	Two-Wire Line	Planar Line
R (Ω/m)	$\dfrac{1}{2\pi\delta\sigma_c}\left[\dfrac{1}{a}+\dfrac{1}{b}\right]$ $(\delta \ll a, c - b)$	$\dfrac{1}{\pi a\delta\sigma_c}$ $(\delta \ll a)$	$\dfrac{2}{w\delta\sigma_c}$ $(\delta \ll t)$
L (H/m)	$\dfrac{\mu}{2\pi}\ln\dfrac{b}{a}$	$\dfrac{\mu}{\pi}\cosh^{-1}\dfrac{d}{2a}$	$\dfrac{\mu d}{w}$
G (S/m)	$\dfrac{2\pi\sigma}{\ln\dfrac{b}{a}}$	$\dfrac{\pi\sigma}{\cosh^{-1}\dfrac{d}{2a}}$	$\dfrac{\sigma w}{d}$
C (F/m)	$\dfrac{2\pi\varepsilon}{\ln\dfrac{b}{a}}$	$\dfrac{\pi\varepsilon}{\cosh^{-1}\dfrac{d}{2a}}$	$\dfrac{\varepsilon w}{d}$ $(w \gg d)$

$*\delta = \dfrac{1}{\sqrt{\pi f \mu_c \sigma_c}}$ = skin depth of the conductor; $\cosh^{-1}\dfrac{d}{2a} \simeq \ln\dfrac{d}{a}$ if $\left[\dfrac{d}{2a}\right]^2 \gg 1$.

2. For each line, the conductors are characterized by σ_c, μ_c, $\varepsilon_c = \varepsilon_o$, and the homogeneous dielectric separating the conductors is characterized by σ, μ, ε.
3. $G \neq 1/R$; R is the ac resistance per unit length of the conductors comprising the line and G is the conductance per unit length due to the dielectric medium separating the conductors.
4. The value of L shown in Table 11.1 is the external inductance per unit length; that is, $L = L_{ext}$. The effects of internal inductance L_{in} $(= R/\omega)$ are negligible as high frequencies at which most communication systems operate.
5. For each line,

$$LC = \mu\varepsilon \qquad \text{and} \qquad \frac{G}{C} = \frac{\sigma}{\varepsilon} \tag{11.1}$$

As a way of preparing for the next section, let us consider how an EM wave propagates through a two-conductor transmission line. For example, consider the coaxial line connecting the generator or source to the load as in Figure 11.4(a). When switch S is closed,

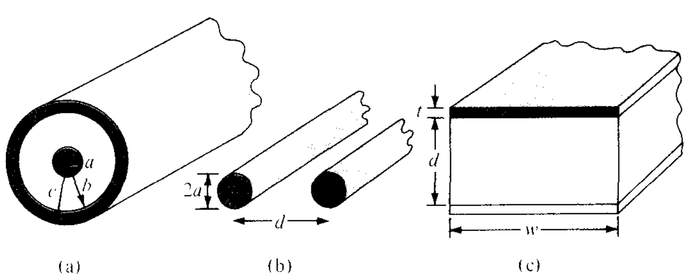


Figure 11.2 Common transmission lines: (a) coaxial line, (b) two-wire line, (c) planar line.

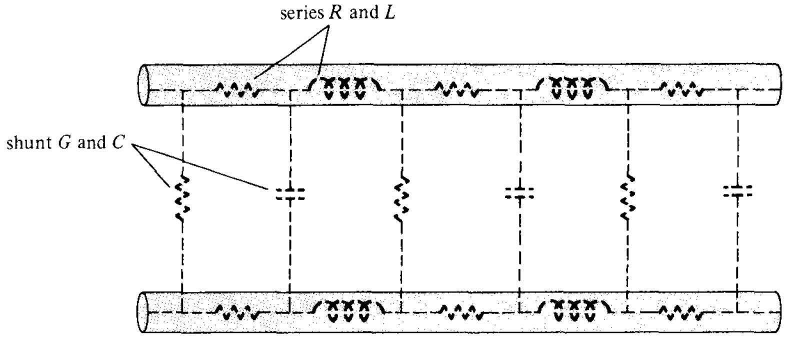
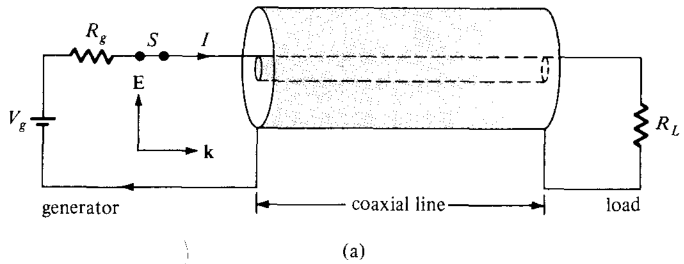


Figure 11.3 Distributed parameters of a two-conductor transmission line.

the inner conductor is made positive with respect to the outer one so that the **E** field is radially outward as in Figure 11.4(b). According to Ampere's law, the **H** field encircles the current carrying conductor as in Figure 11.4(b). The Poynting vector (**E** $\times$ **H**) points along the transmission line. Thus, closing the switch simply establishes a disturbance, which appears as a transverse electromagnetic (TEM) wave propagating along the line. This wave is a nonuniform plane wave and by means of it power is transmitted through the line.



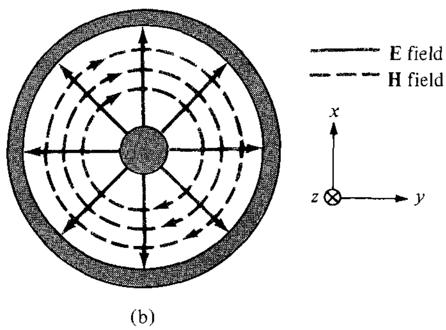


Figure 11.4 (a) Coaxial line connecting the generator to the load; **(b) E** and **H** fields on the coaxial line.

11.3 TRANSMISSION LINE EQUATIONS

As mentioned in the previous section, a two-conductor transmission line supports a TEM wave; that is, the electric and magnetic fields on the line are transverse to the direction of wave propagation. An important property of TEM waves is that the fields **E** and **H** are uniquely related to voltage V and current I, respectively:

$$V = -\int \mathbf{E} \cdot d\mathbf{l}, \qquad I = \oint \mathbf{H} \cdot d\mathbf{l} \tag{11.2}$$

In view of this, we will use circuit quantities V and I in solving the transmission line problem instead of solving field quantities **E** and **H** (i.e., solving Maxwell's equations and boundary conditions). The circuit model is simpler and more convenient.

Let us examine an incremental portion of length Δz of a two-conductor transmission line. We intend to find an equivalent circuit for this line and derive the line equations. From Figure 11.3, we expect the equivalent circuit of a portion of the line to be as in Figure 11.5. The model in Figure 11.5 is in terms of the line parameters R, L, G, and C, and may represent any of the two-conductor lines of Figure 11.3. The model is called the L-type equivalent circuit; there are other possible types (see Problem 11.1). In the model of Figure 11.5, we assume that the wave propagates along the $+z$-direction, from the generator to the load.

By applying Kirchhoff's voltage law to the outer loop of the circuit in Figure 11.5, we obtain

$$V(z, t) = R\,\Delta z\, I(z, t) + L\,\Delta z \frac{\partial I(z, t)}{\partial t} + V(z + \Delta z, t)$$

or

$$-\frac{V(z + \Delta z, t) - V(z, t)}{\Delta z} = R\, I(z, t) + L \frac{\partial I(z, t)}{\partial t} \tag{11.3}$$

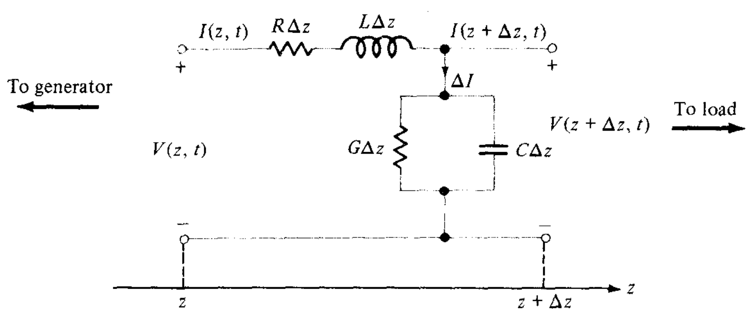


Figure 11.5 L-type equivalent circuit model of a differential length Δz of a two-conductor transmission line.

Taking the limit of eq. (11.3) as $\Delta z \to 0$ leads to

$$\boxed{-\frac{\partial V(z, t)}{\partial z} = RI(z, t) + L\frac{\partial I(z, t)}{\partial t}} \tag{11.4}$$

Similarly, applying Kirchoff's current law to the main node of the circuit in Figure 11.5 gives

$$\begin{aligned} I(z, t) &= I(z + \Delta z, t) + \Delta I \\ &= I(z + \Delta z, t) + G\,\Delta z\, V(z + \Delta z, t) + C\,\Delta z\frac{\partial V(z + \Delta z, t)}{\partial t} \end{aligned}$$

or

$$-\frac{I(z + \Delta z, t) - I(z, t)}{\Delta z} = G\,V(z + \Delta z, t) + C\frac{\partial V(z + \Delta z, t)}{\partial t} \tag{11.5}$$

As $\Delta z \to 0$, eq. (11.5) becomes

$$\boxed{-\frac{\partial I(z, t)}{\partial z} = G\,V(z, t) + C\frac{\partial V(z, t)}{\partial t}} \tag{11.6}$$

If we assume harmonic time dependence so that

$$V(z, t) = \text{Re}\,[V_s(z)\,e^{j\omega t}] \tag{11.7a}$$

$$I(z, t) = \text{Re}\,[I_s(z)\,e^{j\omega t}] \tag{11.7b}$$

where $V_s(z)$ and $I_s(z)$ are the phasor forms of $V(z, t)$ and $I(z, t)$, respectively, eqs. (11.4) and (11.6) become

$$-\frac{dV_s}{dz} = (R + j\omega L)\,I_s \tag{11.8}$$

$$-\frac{dI_s}{dz} = (G + j\omega C)\,V_s \tag{11.9}$$

In the differential eqs. (11.8) and (11.9), V_s and I_s are coupled. To separate them, we take the second derivative of V_s in eq. (11.8) and employ eq. (11.9) so that we obtain

$$\frac{d^2V_s}{dz^2} = (R + j\omega L)(G + j\omega C)\,V_s$$

or

$$\boxed{\frac{d^2V_s}{dz^2} - \gamma^2 V_s = 0} \tag{11.10}$$

where

$$\boxed{\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}} \tag{11.11}$$

By taking the second derivative of I_s in eq. (11.9) and employing eq. (11.8), we get

$$\frac{d^2 I_s}{dz^2} - \gamma^2 I_s = 0 \tag{11.12}$$

We notice that eqs. (11.10) and (11.12) are, respectively, the wave equations for voltage and current similar in form to the wave equations obtained for plane waves in eqs. (10.17) and (10.19). Thus, in our usual notations, γ in eq. (11.11) is the propagation constant (in per meter), α is the attenuation constant (in nepers per meter or decibels[2] per meter), and β is the phase constant (in radians per meter). The wavelength λ and wave velocity u are, respectively, given by

$$\lambda = \frac{2\pi}{\beta} \tag{11.13}$$

$$\boxed{u = \frac{\omega}{\beta} = f\lambda} \tag{11.14}$$

The solutions of the linear homogeneous differential equations (11.10) and (11.12) are similar to Case 2 of Example 6.5, namely,

$$V_s(z) = \underset{\longrightarrow +z}{V_o^+ e^{-\gamma z}} + \underset{-z \longleftarrow}{V_o^- e^{\gamma z}} \tag{11.15}$$

and

$$I_s(z) = \underset{\longrightarrow +z}{I_o^+ e^{-\gamma z}} + \underset{-z \longleftarrow}{I_o^- e^{\gamma z}} \tag{11.16}$$

where V_o^+, V_o^-, I_o^+, and I_o^- are wave amplitudes; the + and − signs, respectively, denote wave traveling along $+z$- and $-z$-directions, as is also indicated by the arrows. Thus, we obtain the instantaneous expression for voltage as

$$\begin{aligned} V(z, t) &= \text{Re}\,[V_s(z)\, e^{j\omega t}] \\ &= V_o^+ e^{-\alpha z} \cos(\omega t - \beta z) + V_o^- e^{\alpha z} \cos(\omega t + \beta z) \end{aligned} \tag{11.17}$$

The **characteristic impedance** Z_o of the line is the ratio of positively traveling voltage wave to current wave at any point on the line.

[2]Recall from eq. (10.35) that 1 Np = 8.686 dB.

Z_o is analogous to η, the intrinsic impedance of the medium of wave propagation. By substituting eqs. (11.15) and (11.16) into eqs. (11.8) and (11.9) and equating coefficients of terms $e^{\gamma z}$ and $e^{-\gamma z}$, we obtain

$$Z_o = \frac{V_o^+}{I_o^+} = -\frac{V_o^-}{I_o^-} = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C} \tag{11.18}$$

or

$$\boxed{Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_o + jX_o} \tag{11.19}$$

where R_o and X_o are the real and imaginary parts of Z_o. R_o should not be mistaken for R—while R is in ohms per meter; R_o is in ohms. The propagation constant γ and the characteristic impedance Z_o are important properties of the line because they both depend on the line parameters R, L, G, and C and the frequency of operation. The reciprocal of Z_o is the characteristic admittance Y_o, that is, $Y_o = 1/Z_o$.

The transmission line considered thus far in this section is the *lossy* type in that the conductors comprising the line are imperfect ($\sigma_c \neq \infty$) and the dielectric in which the conductors are embedded is lossy ($\sigma \neq 0$). Having considered this general case, we may now consider two special cases of lossless transmission line and distortionless line.

A. Lossless Line ($R = 0 = G$)

A **transmission line** is said to be **lossless** if the conductors of the line are perfect ($\sigma_c \approx \infty$) and the dielectric medium separating them is lossless ($\sigma \simeq 0$).

For such a line, it is evident from Table 11.1 that when $\sigma_c \simeq \infty$ and $\sigma \simeq 0$.

$$\boxed{R = 0 = G} \tag{11.20}$$

This is a necessary condition for a line to be lossless. Thus for such a line, eq. (11.20) forces eqs. (11.11), (11.14), and (11.19) to become

$$\alpha = 0, \qquad \gamma = j\beta = j\omega\sqrt{LC} \tag{11.21a}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda \tag{11.21b}$$

$$X_o = 0, \qquad Z_o = R_o = \sqrt{\frac{L}{C}} \tag{11.21c}$$

B. Distortionless Line ($R/L = G/C$)

A signal normally consists of a band of frequencies; wave amplitudes of different frequency components will be attenuated differently in a lossy line as α is frequency dependent. This results in distortion.

> A **distortionless line** is one in which the attenuation constant α is frequency independent while the phase constant β is linearly dependent on frequency.

From the general expression for α and β [in eq. (11.11)], a distortionless line results if the line parameters are such that

$$\boxed{\frac{R}{L} = \frac{G}{C}} \tag{11.22}$$

Thus, for a distortionless line,

$$\begin{aligned}\gamma &= \sqrt{RG\left(1 + \frac{j\omega L}{R}\right)\left(1 + \frac{j\omega C}{G}\right)} \\ &= \sqrt{RG}\left(1 + \frac{j\omega C}{G}\right) = \alpha + j\beta\end{aligned}$$

or

$$\alpha = \sqrt{RG}, \qquad \beta = \omega\sqrt{LC} \tag{11.23a}$$

showing that α does not depend on frequency whereas β is a linear function of frequency. Also

$$Z_o = \sqrt{\frac{R(1 + j\omega L/R)}{G(1 + j\omega C/G)}} = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} = R_o + jX_o$$

or

$$R_o = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}, \qquad X_o = 0 \tag{11.23b}$$

and

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda \tag{11.23c}$$

Note that

1. The phase velocity is independent of frequency because the phase constant β linearly depends on frequency. We have shape distortion of signals unless α and u are independent of frequency.
2. u and Z_o remain the same as for lossless lines.

TABLE 11.2 Transmission Line Characteristics

Case	Propagation Constant $\gamma = \alpha + j\beta$	Characteristic Impedance $Z_o = R_o + jX_o$
General	$\sqrt{(R + j\omega L)(G + j\omega C)}$	$\sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$
Lossless	$0 + j\omega\sqrt{LC}$	$\sqrt{\dfrac{L}{C}} + j0$
Distortionless	$\sqrt{RG} + j\omega\sqrt{LC}$	$\sqrt{\dfrac{L}{C}} + j0$

3. A lossless line is also a distortionless line, but a distortionless line is not necessarily lossless. Although lossless lines are desirable in power transmission, telephone lines are required to be distortionless.

A summary of our discussion is in Table 11.2. For the greater part of our analysis, we shall restrict our discussion to lossless transmission lines.

EXAMPLE 11.1

An air line has characteristic impedance of 70 Ω and phase constant of 3 rad/m at 100 MHz. Calculate the inductance per meter and the capacitance per meter of the line.

Solution:

An air line can be regarded as a lossless line since $\sigma \simeq 0$. Hence

$$R = 0 = G \qquad \text{and} \qquad \alpha = 0$$

$$Z_o = R_o = \sqrt{\frac{L}{C}} \tag{11.1.1}$$

$$\beta = \omega\sqrt{LC} \tag{11.1.2}$$

Dividing eq. (11.1.1) by eq. (11.1.2) yields

$$\frac{R_o}{\beta} = \frac{1}{\omega C}$$

or

$$C = \frac{\beta}{\omega R_o} = \frac{3}{2\pi \times 100 \times 10^6(70)} = 68.2 \text{ pF/m}$$

From eq. (11.1.1),

$$L = R_o^2 C = (70)^2(68.2 \times 10^{-12}) = 334.2 \text{ nH/m}$$

PRACTICE EXERCISE 11.1

A transmission line operating at 500 MHz has $Z_o = 80\ \Omega$, $\alpha = 0.04$ Np/m, $\beta =$ 1.5 rad/m. Find the line parameters R, L, G, and C.

Answer: 3.2 Ω/m, 38.2 nH/m, 5×10^{-4} S/m, 5.97 pF/m.

EXAMPLE 11.2

A distortionless line has $Z_o = 60\ \Omega$, $\alpha = 20$ mNp/m, $u = 0.6c$, where c is the speed of light in a vacuum. Find R, L, G, C, and λ at 100 MHz.

Solution:

For a distortionless line,

$$RC = GL \qquad \text{or} \qquad G = \frac{RC}{L}$$

and hence

$$Z_o = \sqrt{\frac{L}{C}} \tag{11.2.1}$$

$$\alpha = \sqrt{RG} = R\sqrt{\frac{C}{L}} = \frac{R}{Z_o} \tag{11.2.2a}$$

or

$$R = \alpha Z_o \tag{11.2.2b}$$

But

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \tag{11.2.3}$$

From eq. (11.2.2b),

$$R = \alpha Z_o = (20 \times 10^{-3})(60) = 1.2\ \Omega/\text{m}$$

Dividing eq. (11.2.1) by eq. (11.2.3) results in

$$L = \frac{Z_o}{u} = \frac{60}{0.6\,(3 \times 10^8)} = 333\ \text{nH/m}$$

From eq. (11.2.2a),

$$G = \frac{\alpha^2}{R} = \frac{400 \times 10^{-6}}{1.2} = 333\ \mu\text{S/m}$$

Multiplying eqs. (11.2.1) and (11.2.3) together gives

$$uZ_o = \frac{1}{C}$$

or

$$C = \frac{1}{uZ_o} = \frac{1}{0.6\,(3 \times 10^8)\,60} = 92.59 \text{ pF/m}$$

$$\lambda = \frac{u}{f} = \frac{0.6\,(3 \times 10^8)}{10^8} = 1.8 \text{ m}$$

PRACTICE EXERCISE 11.2

A telephone line has $R = 30\ \Omega/\text{km}$, $L = 100$ mH/km, $G = 0$, and $C = 20\ \mu\text{F/km}$. At $f = 1$ kHz, obtain:

(a) The characteristic impedance of the line

(b) The propagation constant

(c) The phase velocity

Answer: (a) $70.75\underline{/-1.367^\circ}\ \Omega$, (b) $2.121 \times 10^{-4} + j8.888 \times 10^{-3}$/m, (c) $7.069 \times$ 105 m/s.

11.4 INPUT IMPEDANCE, SWR, AND POWER

Consider a transmission line of length ℓ, characterized by γ and Z_o, connected to a load Z_L as shown in Figure 11.6. Looking into the line, the generator sees the line with the load as an input impedance Z_{in}. It is our intention in this section to determine the input impedance, the standing wave ratio (SWR), and the power flow on the line.

Let the transmission line extend from $z = 0$ at the generator to $z = \ell$ at the load. First of all, we need the voltage and current waves in eqs. (11.15) and (11.16), that is

$$V_s(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z} \tag{11.24}$$

$$I_s(z) = \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{\gamma z} \tag{11.25}$$

where eq. (11.18) has been incorporated. To find V_o^+ and V_o^-, the terminal conditions must be given. For example, if we are given the conditions at the input, say

$$V_o = V(z = 0), \qquad I_o = I(z = 0) \tag{11.26}$$

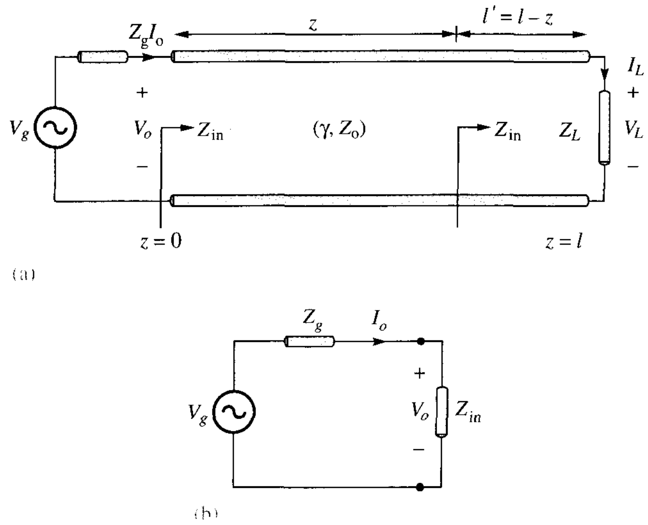


Figure 11.6 **(a)** Input impedance due to a line terminated by a load; **(b)** equivalent circuit for finding V_o and I_o in terms of Z_{in} at the input.

substituting these into eqs. (11.24) and (11.25) results in

$$V_o^+ = \frac{1}{2}(V_o + Z_o I_o) \tag{11.27a}$$

$$V_o^- = \frac{1}{2}(V_o - Z_o I_o) \tag{11.27b}$$

If the input impedance at the input terminals is Z_{in}, the input voltage V_o and the input current I_o are easily obtained from Figure 11.6(b) as

$$V_o = \frac{Z_{in}}{Z_{in} + Z_g} V_g, \qquad I_o = \frac{V_g}{Z_{in} + Z_g} \tag{11.28}$$

On the other hand, if we are given the conditions at the load, say

$$V_L = V(z = \ell), \qquad I_L = I(z = \ell) \tag{11.29}$$

Substituting these into eqs. (11.24) and (11.25) gives

$$V_o^+ = \frac{1}{2}(V_L + Z_o I_L)e^{\gamma\ell} \tag{11.30a}$$

$$V_o^- = \frac{1}{2}(V_L - Z_o I_L)e^{-\gamma\ell} \tag{11.30b}$$

Next, we determine the input impedance $Z_{in} = V_s(z)/I_s(z)$ at any point on the line. At the generator, for example, eqs. (11.24) and (11.25) yield

$$Z_{in} = \frac{V_s(z)}{I_s(z)} = \frac{Z_o(V_o^+ + V_o^-)}{V_o^+ - V_o^-} \tag{11.31}$$

Substituting eq. (11.30) into (11.31) and utilizing the fact that

$$\frac{e^{\gamma\ell} + e^{-\gamma\ell}}{2} = \cosh \gamma\ell, \qquad \frac{e^{\gamma\ell} - e^{-\gamma\ell}}{2} = \sinh \gamma\ell \tag{11.32a}$$

or

$$\tanh \gamma\ell = \frac{\sinh \gamma\ell}{\cosh \gamma\ell} = \frac{e^{\gamma\ell} - e^{-\gamma\ell}}{e^{\gamma\ell} + e^{-\gamma\ell}} \tag{11.32b}$$

we get

$$\boxed{Z_{in} = Z_o \left[\frac{Z_L + Z_o \tanh \gamma\ell}{Z_o + Z_L \tanh \gamma\ell}\right]} \qquad \text{(lossy)} \tag{11.33}$$

Although eq. (11.33) has been derived for the input impedance Z_{in} at the generation end, it is a general expression for finding Z_{in} at any point on the line. To find Z_{in} at a distance ℓ' from the load as in Figure 11.6(a), we replace ℓ by ℓ'. A formula for calculating the hyperbolic tangent of a complex number, required in eq. (11.33), is found in Appendix A.3.

For a lossless line, $\gamma = j\beta$, $\tanh j\beta\ell = j \tan \beta\ell$, and $Z_o = R_o$, so eq. (11.33) becomes

$$\boxed{Z_{in} = Z_o \left[\frac{Z_L + jZ_o \tan \beta\ell}{Z_o + jZ_L \tan \beta\ell}\right]} \qquad \text{(lossless)} \tag{11.34}$$

showing that the input impedance varies periodically with distance ℓ from the load. The quantity $\beta\ell$ in eq. (11.34) is usually referred to as the *electrical length* of the line and can be expressed in degrees or radians.

We now define Γ_L as the *voltage reflection coefficient* (at the load). Γ_L is the ratio of the voltage reflection wave to the incident wave at the load, that is,

$$\Gamma_L = \frac{V_o^- e^{\gamma\ell}}{V_o^+ e^{-\gamma\ell}} \tag{11.35}$$

Substituting V_o^- and V_o^+ in eq. (11.30) into eq. (11.35) and incorporating $V_L = Z_L I_L$ gives

$$\boxed{\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}} \tag{11.36}$$

The **voltage reflection coefficient** at any point on the line is the ratio of the magnitude of the reflected voltage wave to that of the incident wave.

That is,

$$\Gamma(z) = \frac{V_o^- e^{\gamma z}}{V_o^+ e^{-\gamma z}} = \frac{V_o^-}{V_o^+} e^{2\gamma z}$$

But $z = \ell - \ell'$. Substituting and combining with eq. (11.35), we get

$$\Gamma(z) = \frac{V_o^-}{V_o^+} e^{2\gamma\ell} e^{-2\gamma\ell'} = \Gamma_L e^{-2\gamma\ell'} \tag{11.37}$$

The **current reflection coefficient** at any point on the line is negative of the voltage reflection coefficient at that point.

Thus, the current reflection coefficient at the load is $I_o^- e^{\gamma\ell}/I_o^+ e^{-\gamma\ell} = -\Gamma_L$.

Just as we did for plane waves, we define the *standing wave ratio s* (otherwise denoted by SWR) as

$$\boxed{s = \frac{V_{\max}}{V_{\min}} = \frac{I_{\max}}{I_{\min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}} \tag{11.38}$$

It is easy to show that $I_{\max} = V_{\max}/Z_o$ and $I_{\min} = V_{\min}/Z_o$. The input impedance Z_{in} in eq. (11.34) has maxima and minima that occur, respectively, at the maxima and minima of the voltage and current standing wave. It can also be shown that

$$\boxed{|Z_{in}|_{\max} = \frac{V_{\max}}{I_{\min}} = sZ_o} \tag{11.39a}$$

and

$$\boxed{|Z_{in}|_{\min} = \frac{V_{\min}}{I_{\max}} = \frac{Z_o}{s}} \tag{11.39b}$$

As a way of demonstrating these concepts, consider a lossless line with characteristic impedance of $Z_o = 50\ \Omega$. For the sake of simplicity, we assume that the line is terminated in a pure resistive load $Z_L = 100\ \Omega$ and the voltage at the load is 100 V (rms). The conditions on the line are displayed in Figure 11.7. Note from the figure that conditions on the line repeat themselves every half wavelength.

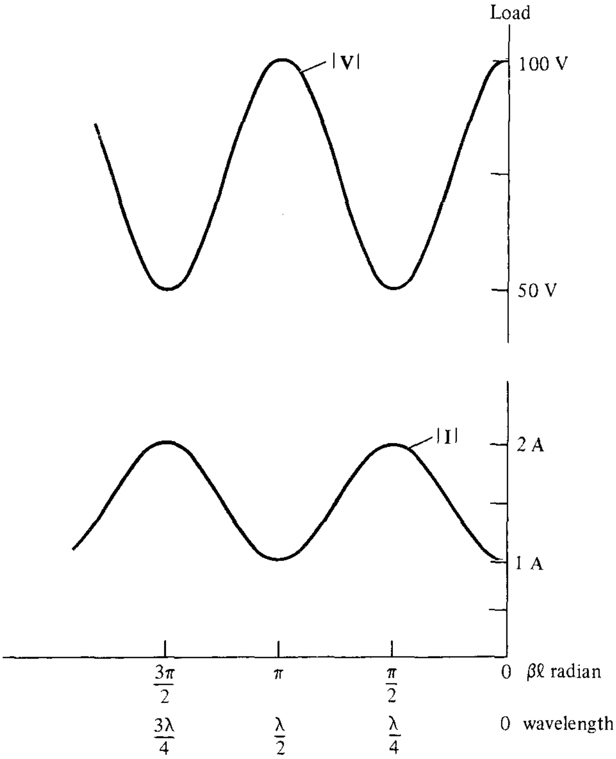


Figure 11.7 Voltage and current wave patterns on a lossless line terminated by a resistive load.

As mentioned at the beginning of this chapter, a transmission is used in transferring power from the source to the load. The average input power at a distance ℓ from the load is given by an equation similar to eq. (10.68); that is

$$P_{\text{ave}} = \frac{1}{2}\,\text{Re}\,[V_s(\ell)I_s^*(\ell)]$$

where the factor $\frac{1}{2}$ is needed since we are dealing with the peak values instead of the rms values. Assuming a lossless line, we substitute eqs. (11.24) and (11.25) to obtain

$$\begin{aligned} P_{\text{ave}} &= \frac{1}{2}\,\text{Re}\left[V_o^+(e^{j\beta\ell} + \Gamma e^{-j\beta\ell})\frac{V^{+*}}{Z_o}(e^{-j\beta\ell} - \Gamma^* e^{j\beta\ell})\right] \\ &= \frac{1}{2}\,\text{Re}\left[\frac{|V_o^+|^2}{Z_o}(1 - |\Gamma|^2 + \Gamma e^{-2j\beta\ell} - \Gamma^* e^{2j\beta\ell})\right] \end{aligned}$$

Since the last two terms are purely imaginary, we have

$$\boxed{P_{\text{ave}} = \frac{|V_o^+|^2}{2Z_o}(1 - |\Gamma|^2)} \tag{11.40}$$

The first term is the incident power P_i, while the second term is the reflected power P_r. Thus eq. (11.40) may be written as

$$P_t = P_i - P_r$$

where P_t is the input or transmitted power and the negative sign is due to the negative-going wave since we take the reference direction as that of the voltage/current traveling toward the right. We should notice from eq. (11.40) that the power is constant and does not depend on ℓ since it is a lossless line. Also, we should notice that maximum power is delivered to the load when $\Gamma = 0$, as expected.

We now consider special cases when the line is connected to load $Z_L = 0$, $Z_L = \infty$, and $Z_L = Z_o$. These special cases can easily be derived from the general case.

A. Shorted Line ($Z_L = 0$)

For this case, eq. (11.34) becomes

$$Z_{sc} = Z_{in}\bigg|_{Z_L=0} = jZ_o \tan \beta\ell \tag{11.41a}$$

Also,

$$\Gamma_L = -1, \qquad s = \infty \tag{11.41b}$$

We notice from eq. (11.41a) that Z_{in} is a pure reactance, which could be capacitive or inductive depending on the value of ℓ. The variation of Z_{in} with ℓ is shown in Figure 11.8(a).

B. Open-Circuited Line ($Z_L = \infty$)

In this case, eq. (11.34) becomes

$$Z_{oc} = \lim_{Z_L \to \infty} Z_{in} = \frac{Z_o}{j \tan \beta\ell} = -jZ_o \cot \beta\ell \tag{11.42a}$$

and

$$\Gamma_L = 1, \qquad s = \infty \tag{11.42b}$$

The variation of Z_{in} with ℓ is shown in Figure 11.8(b). Notice from eqs. (11.41a) and (11.42a) that

$$Z_{sc}Z_{oc} = Z_o^2 \tag{11.43}$$

C. Matched Line ($Z_L = Z_o$)

This is the most desired case from the practical point of view. For this case, eq. (11.34) reduces to

$$Z_{in} = Z_o \tag{11.44a}$$

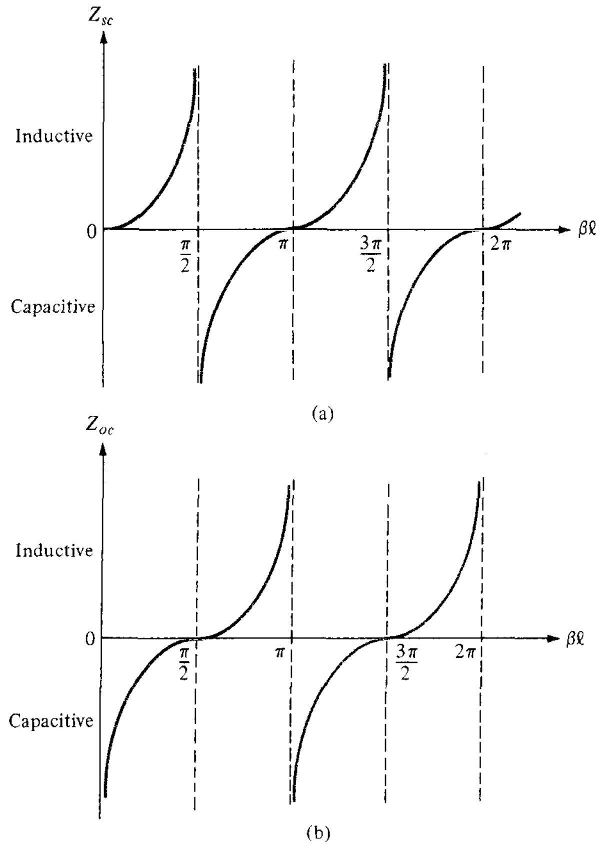


Figure 11.8 Input impedance of a lossless line: **(a)** when shorted, **(b)** when open.

and

$$\Gamma_L = 0, \qquad s = 1 \tag{11.44b}$$

that is, $V_o^- = 0$, the whole wave is transmitted and there is no reflection. The incident power is fully absorbed by the load. Thus maximum power transfer is possible when a transmission line is matched to the load.

EXAMPLE 11.3

A certain transmission line operating at $\omega = 10^6$ rad/s has $\alpha = 8$ dB/m, $\beta = 1$ rad/m, and $Z_o = 60 + j40\ \Omega$, and is 2 m long. If the line is connected to a source of $10\underline{/0^\circ}$ V, $Z_g = 40\ \Omega$ and terminated by a load of $20 + j50\ \Omega$, determine

(a) The input impedance

(b) The sending-end current

(c) The current at the middle of the line

Solution:

(a) Since 1 Np = 8.686 dB,

$$\alpha = \frac{8}{8.686} = 0.921 \text{ Np/m}$$

$$\gamma = \alpha + j\beta = 0.921 + j1 \text{ /m}$$

$$\gamma\ell = 2(0.921 + j1) = 1.84 + j2$$

Using the formula for $\tanh(x + jy)$ in Appendix A.3, we obtain

$$\tanh \gamma\ell = 1.033 - j0.03929$$

$$\begin{aligned} Z_{\text{in}} &= Z_o \left(\frac{Z_L + Z_o \tanh \gamma\ell}{Z_o + Z_L \tanh \gamma\ell} \right) \\ &= (60 + j40) \left[\frac{20 + j50 + (60 + j40)(1.033 - j0.03929)}{60 + j40 + (20 + j50)(1.033 - j0.03929)} \right] \\ Z_{\text{in}} &= 60.25 + j38.79 \ \Omega \end{aligned}$$

(b) The sending-end current is $I(z = 0) = I_o$. From eq. (11.28),

$$\begin{aligned} I(z = 0) &= \frac{V_g}{Z_{\text{in}} + Z_g} = \frac{10}{60.25 + j38.79 + 40} \\ &= 93.03\angle{-21.15^\circ} \text{ mA} \end{aligned}$$

(c) To find the current at any point, we need V_o^+ and V_o^-. But

$$I_o = I(z = 0) = 93.03\angle{-21.15^\circ} \text{ mA}$$

$$V_o = Z_{\text{in}} I_o = (71.66\angle{32.77^\circ})(0.09303\angle{-21.15^\circ}) = 6.667\angle{11.62^\circ} \text{ V}$$

From eq. (11.27),

$$\begin{aligned} V_o^+ &= \frac{1}{2}(V_o + Z_o I_o) \\ &= \frac{1}{2}[6.667\angle{11.62^\circ} + (60 + j40)(0.09303\angle{-21.15^\circ})] = 6.687\angle{12.08^\circ} \end{aligned}$$

$$V_o^- = \frac{1}{2}(V_o - Z_o I_o) = 0.0518\angle{260^\circ}$$

At the middle of the line, $z = \ell/2$, $\gamma z = 0.921 + j1$. Hence, the current at this point is

$$\begin{aligned} I_s(z = \ell/2) &= \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{\gamma z} \\ &= \frac{(6.687e^{j12.08^\circ})e^{-0.921 - j1}}{60 + j40} - \frac{(0.0518e^{j260^\circ})e^{0.921 + j1}}{60 + j40} \end{aligned}$$

Note that $j1$ is in radians and is equivalent to $j57.3°$. Thus,

$$\begin{aligned} I_s(z = \ell/2) &= \frac{6.687e^{j12.08°}e^{-0.921}e^{-j57.3°}}{72.1e^{j33.69°}} - \frac{0.0518e^{j260°}e^{0.921}e^{j57.3°}}{72.1e^{33.69°}} \\ &= 0.0369e^{-j78.91°} - 0.001805e^{j283.61°} \\ &= 6.673 - j34.456 \text{ mA} \\ &= 35.10\underline{/281°} \text{ mA} \end{aligned}$$

PRACTICE EXERCISE 11.3

A 40-m-long transmission line shown in Figure 11.9 has $V_g = 15\underline{/0°}\ V_{rms}$, $Z_o = 30 + j60\ \Omega$, and $V_L = 5\underline{/-48°}\ V_{rms}$. If the line is matched to the load, calculate:

(a) The input impedance Z_{in}

(b) The sending-end current I_{in} and voltage V_{in}

(c) The propagation constant γ

Answer: (a) $30 + j60\ \Omega$, (b) $0.112\underline{/-63.43°}$ A, $7.5\underline{/0°}\ V_{rms}$, (c) $0.0101 + j0.2094$ /m.

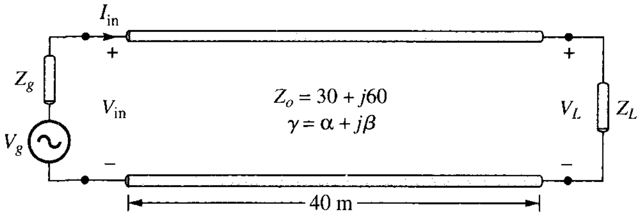


Figure 11.9 For Practice Exercise 11.3.

11.5 THE SMITH CHART

Prior to the advent of digital computers and calculators, engineers developed all sorts of aids (tables, charts, graphs, etc.) to facilitate their calculations for design and analysis. To reduce the tedious manipulations involved in calculating the characteristics of transmission lines, graphical means have been developed. The Smith chart[3] is the most commonly used of the graphical techniques. It is basically a graphical indication of the impedance of a transmission line as one moves along the line. It becomes easy to use after a small amount of experience. We will first examine how the Smith chart is constructed and later

[3]Devised by Phillip H. Smith in 1939. See P. H. Smith, "Transmission line calculator." *Electronics,* vol. 12, pp. 29–31, 1939 and P. H. Smith, "An improved transmission line calculator." *Electronics,* vol. 17, pp. 130–133, 318–325, 1944.

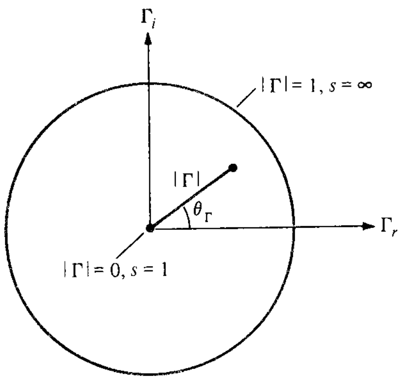


Figure 11.10 Unit circle on which the Smith chart is constructed.

employ it in our calculations of transmission line characteristics such as Γ_L, s, and Z_{in}. We will assume that the transmission line to which the Smith chart will be applied is lossless ($Z_o = R_o$) although this is not fundamentally required.

The Smith chart is constructed within a circle of unit radius ($|\Gamma| \leq 1$) as shown in Figure 11.10. The construction of the chart is based on the relation in eq. (11.36)[4]; that is,

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} \tag{11.45}$$

or

$$\Gamma = |\Gamma| \underline{/\theta_\Gamma} = \Gamma_r + j\Gamma_i \tag{11.46}$$

where Γ_r and Γ_i are the real and imaginary parts of the reflection coefficient Γ.

Instead of having separate Smith charts for transmission lines with different characteristic impedances such as $Z_o = 60$, 100, and 120 Ω, we prefer to have just one that can be used for any line. We achieve this by using a normalized chart in which all impedances are normalized with respect to the characteristic impedance Z_o of the particular line under consideration. For the load impedance Z_L, for example, the *normalized impedance* z_L is given by

$$z_L = \frac{Z_L}{Z_o} = r + jx \tag{11.47}$$

Substituting eq. (11.47) into eqs. (11.45) and (11.46) gives

$$\Gamma = \Gamma_r + j\Gamma_i = \frac{z_L - 1}{z_L + 1} \tag{11.48a}$$

or

$$z_L = r + jx = \frac{(1 + \Gamma_r) + j\Gamma_i}{(1 - \Gamma_r) - j\Gamma_i} \tag{11.48b}$$

[4]Whenever a subscript is not attached to Γ, we simply mean voltage reflection coefficient at the load ($\Gamma_L = \Gamma$).

Normalizing and equating components, we obtain

$$r = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2} \tag{11.49a}$$

$$x = \frac{2\,\Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2} \tag{11.49b}$$

Rearranging terms in eq. (11.49) leads to

$$\boxed{\left[\Gamma_r - \frac{r}{1 + r}\right]^2 + \Gamma_i^2 = \left[\frac{1}{1 + r}\right]^2} \tag{11.50}$$

and

$$\boxed{[\Gamma_r - 1]^2 + \left[\Gamma_i - \frac{1}{x}\right]^2 = \left[\frac{1}{x}\right]^2} \tag{11.51}$$

Each of eqs. (11.50) and (11.51) is similar to

$$(x - h)^2 + (y - k)^2 = a^2 \tag{11.52}$$

which is the general equation of a circle of radius a, centered at (h, k). Thus eq. (11.50) is an *r-circle (resistance circle)* with

$$\text{center at } (\Gamma_r, \Gamma_i) = \left(\frac{r}{1 + r}, 0\right) \tag{11.53a}$$

$$\text{radius} = \frac{1}{1 + r} \tag{11.53b}$$

For typical values of the normalized resistance r, the corresponding centers and radii of the r-circles are presented in Table 11.3. Typical examples of the r-circles based on the data in

TABLE 11.3 Radii and Centers of r-Circles for Typical Values of r

Normalized Resistance (r)	**Radius** $\left(\frac{1}{1+r}\right)$	**Center** $\left(\frac{r}{1+r}, 0\right)$
0	1	(0, 0)
1/2	2/3	(1/3, 0)
1	1/2	(1/2, 0)
2	1/3	(2/3, 0)
5	1/6	(5/6, 0)
∞	0	(1, 0)

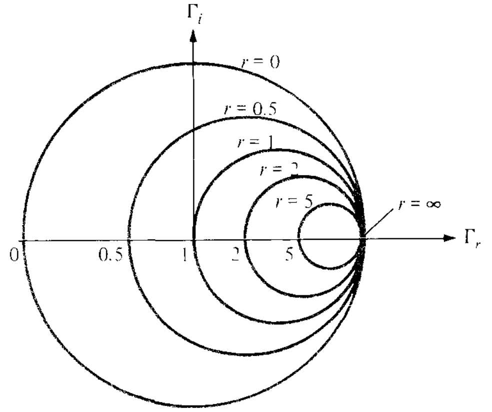


Figure 11.11 Typical r-circles for $r = 0, 0.5, 1, 2, 5, \infty$.

Table 11.3 are shown in Figure 11.11. Similarly, eq. (11.51) is an *x-circle (reactance circle)* with

$$\text{center at } (\Gamma_r, \Gamma_i) = \left(1, \frac{1}{x}\right) \tag{11.54a}$$

$$\text{radius} = \frac{1}{x} \tag{11.54b}$$

Table 11.4 presents centers and radii of the x-circles for typical values of x, and Figure 11.12 shows the corresponding plots. Notice that while r is always positive, x can be positive (for inductive impedance) or negative (for capacitive impedance).

If we superpose the r-circles and x-circles, what we have is the Smith chart shown in Figure 11.13. On the chart, we locate a normalized impedance $z = 2 + j$, for example, as the point of intersection of the $r = 2$ circle and the $x = 1$ circle. This is point P_1 in Figure 11.13. Similarly, $z = 1 - j0.5$ is located at P_2, where the $r = 1$ circle and the $x = -0.5$ circle intersect.

Apart from the r- and x-circles (shown on the Smith chart), we can draw the *s-circles* or *constant standing-wave-ratio circles* (always not shown on the Smith chart), which are centered at the origin with s varying from 1 to ∞. The value of the standing wave ratio s is

TABLE 11.4 Radii and Centers of x-Circles for Typical Value of x

Normalized Reactance (x)	**Radius $\left(\frac{1}{x}\right)$**	**Center $\left(1, \frac{1}{x}\right)$**
0	∞	$(1, \infty)$
$\pm 1/2$	2	$(1, \pm 2)$
± 1	1	$(1, \pm 1)$
± 2	1/2	$(1, \pm 1/2)$
± 5	1/5	$(1, \pm 1/5)$
$\pm \infty$	0	$(1, 0)$

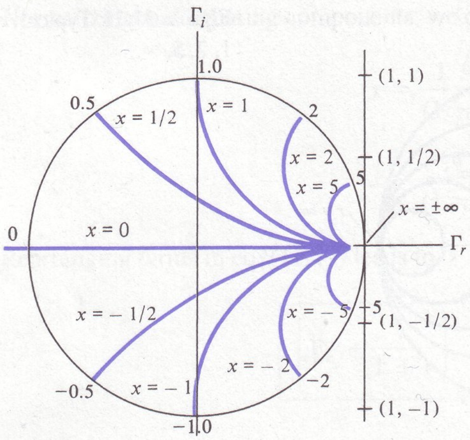


Figure 11.12 Typical x-circles for $x = 0, \pm 1/2, \pm 1, \pm 2, \pm 5, \pm\infty$.

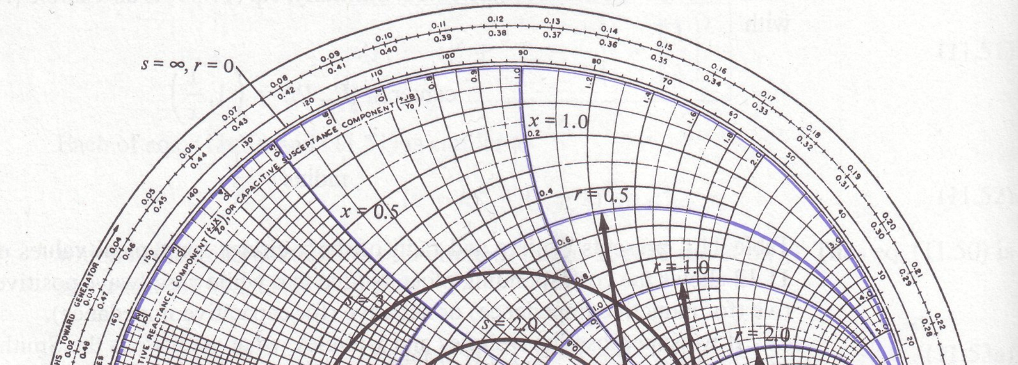


Figure 11.13 Illustration of the r-, x-, and s-circles on the Smith chart.

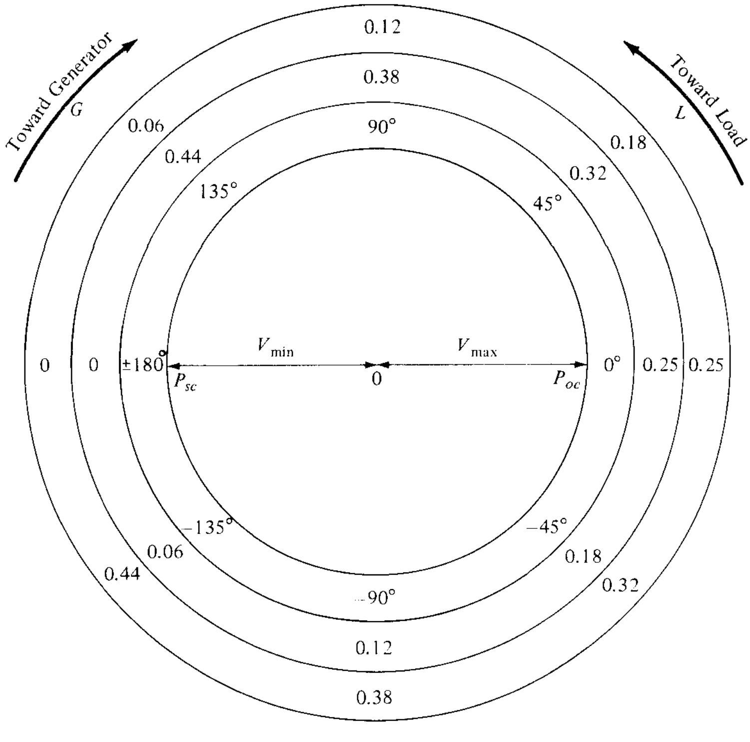
determined by locating where an s-circle crosses the Γ_r axis. Typical examples of s-circles for $s = 1, 2, 3$, and ∞ are shown in Figure 11.13. Since $|\Gamma|$ and s are related according to eq. (11.38), the s-circles are sometimes referred to as $|\Gamma|$-circles with $|\Gamma|$ varying linearly from 0 to 1 as we move away from the center O toward the periphery of the chart while s varies nonlinearly from 1 to ∞.

The following points should be noted about the Smith chart:

1. At point P_{sc} on the chart $r = 0, x = 0$; that is, $Z_L = 0 + j0$ showing that P_{sc} represents a short circuit on the transmission line. At point P_{oc}, $r = \infty$ and $x = \infty$, or $Z_L = \infty + j\infty$, which implies that P_{oc} corresponds to an open circuit on the line. Also at P_{oc}, $r = 0$ and $x = 0$, showing that P_{oc} is another location of a short circuit on the line.
2. A complete revolution (360°) around the Smith chart represents a distance of $\lambda/2$ on the line. Clockwise movement on the chart is regarded as moving toward the generator (or away from the load) as shown by the arrow G in Figure 11.14(a) and (b). Similarly, counterclockwise movement on the chart corresponds to moving toward the load (or away from the generator) as indicated by the arrow L in Figure 11.14. Notice from Figure 11.14(b) that at the load, moving toward the load does not make sense (because we are already at the load). The same can be said of the case when we are at the generator end.
3. There are three scales around the periphery of the Smith chart as illustrated in Figure 11.14(a). The three scales are included for the sake of convenience but they are actually meant to serve the same purpose; one scale should be sufficient. The scales are used in determining the distance from the load or generator in degrees or wavelengths. The outermost scale is used to determine the distance on the line from the generator end in terms of wavelengths, and the next scale determines the distance from the load end in terms of wavelengths. The innermost scale is a protractor (in degrees) and is primarily used in determining θ_Γ; it can also be used to determine the distance from the load or generator. Since a $\lambda/2$ distance on the line corresponds to a movement of 360° on the chart, λ *distance on the line corresponds to a 720° movement on the chart.*

$$\boxed{\lambda \rightarrow 720^\circ} \tag{11.55}$$

   Thus we may ignore the other outer scales and use the protractor (the innermost scale) for all our θ_Γ and distance calculations.
4. V_{max} occurs where $Z_{in,max}$ is located on the chart [see eq. (11.39a)], and that is on the positive Γ_r axis or on OP_{oc} in Figure 11.14(a). V_{min} is located at the same point where we have $Z_{in,min}$ on the chart; that is, on the negative Γ_r axis or on OP_{sc} in Figure 11.14(a). Notice that V_{max} and V_{min} (or $Z_{in,max}$ and $Z_{in,min}$) are $\lambda/4$ (or 180°) apart.
5. The Smith chart is used both as impedance chart and admittance chart ($Y = 1/Z$). As admittance chart (normalized impedance $y = Y/Y_o = g + jb$), the g- and b-circles correspond to r- and x-circles, respectively.



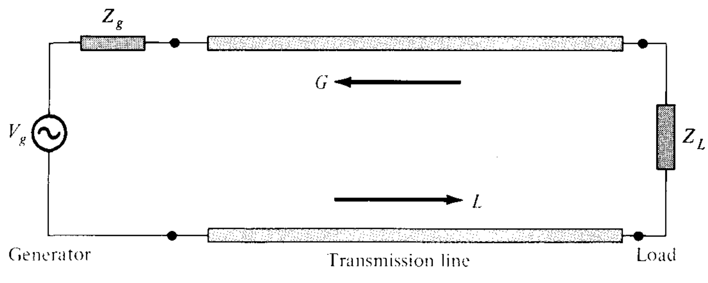


Figure 11.14 (a) Smith chart illustrating scales around the periphery and movements around the chart, **(b)** corresponding movements along the transmission line.

Based on these important properties, the Smith chart may be used to determine, among other things, (a) $\Gamma = |\Gamma|\underline{/\theta_\Gamma}$ and s; (b) Z_{in} or Y_{in}; and (c) the locations of V_{max} and V_{min} provided that we are given Z_o, Z_L, and the length of the line. Some examples will clearly show how we can do all these and much more with the aid of the Smith chart, a compass, and a plain straightedge.

EXAMPLE 11.4

A 30-m-long lossless transmission line with $Z_o = 50\ \Omega$ operating at 2 MHz is terminated with a load $Z_L = 60 + j40\ \Omega$. If $u = 0.6c$ on the line, find

(a) The reflection coefficient Γ

(b) The standing wave ratio s

(c) The input impedance

Solution:

This problem will be solved with and without using the Smith chart.

Method 1: (Without the Smith chart)

(a) $$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{60 + j40 - 50}{50 + j40 + 50} = \frac{10 + j40}{110 + j40}$$
$$= 0.3523\underline{/56^\circ}$$

(b) $$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.3523}{1 - 0.3523} = 2.088$$

(c) Since $u = \omega/\beta$, or $\beta = \omega/u$,

$$\beta\ell = \frac{\omega\ell}{u} = \frac{2\pi\,(2 \times 10^6)(30)}{0.6\,(3 \times 10^8)} = \frac{2\pi}{3} = 120^\circ$$

Note that $\beta\ell$ is the electrical length of the line.

$$Z_{in} = Z_o \left[\frac{Z_L + jZ_o \tan \beta\ell}{Z_o + jZ_L \tan \beta\ell} \right]$$
$$= \frac{50\,(60 + j40 + j50 \tan 120^\circ)}{[50 + j(60 + j40) \tan 120^\circ]}$$
$$= \frac{50\,(6 + j4 - j5\sqrt{3})}{(5 + 4\sqrt{3} - j6\sqrt{3})} = 24.01\underline{/3.22^\circ}$$
$$= 23.97 + j1.35\ \Omega$$

Method 2: (Using the Smith chart).

(a) Calculate the normalized load impedance

$$z_L = \frac{Z_L}{Z_o} = \frac{60 + j40}{50}$$
$$= 1.2 + j0.8$$

Locate z_L on the Smith chart of Figure 11.15 at point P where the $r = 1.2$ circle and the $x = 0.8$ circle meet. To get Γ at z_L, extend OP to meet the $r = 0$ circle at Q and measure OP and OQ. Since OQ corresponds to $|\Gamma| = 1$, then at P,

$$|\Gamma| = \frac{OP}{OQ} = \frac{3.2\text{ cm}}{9.1\text{ cm}} = 0.3516$$

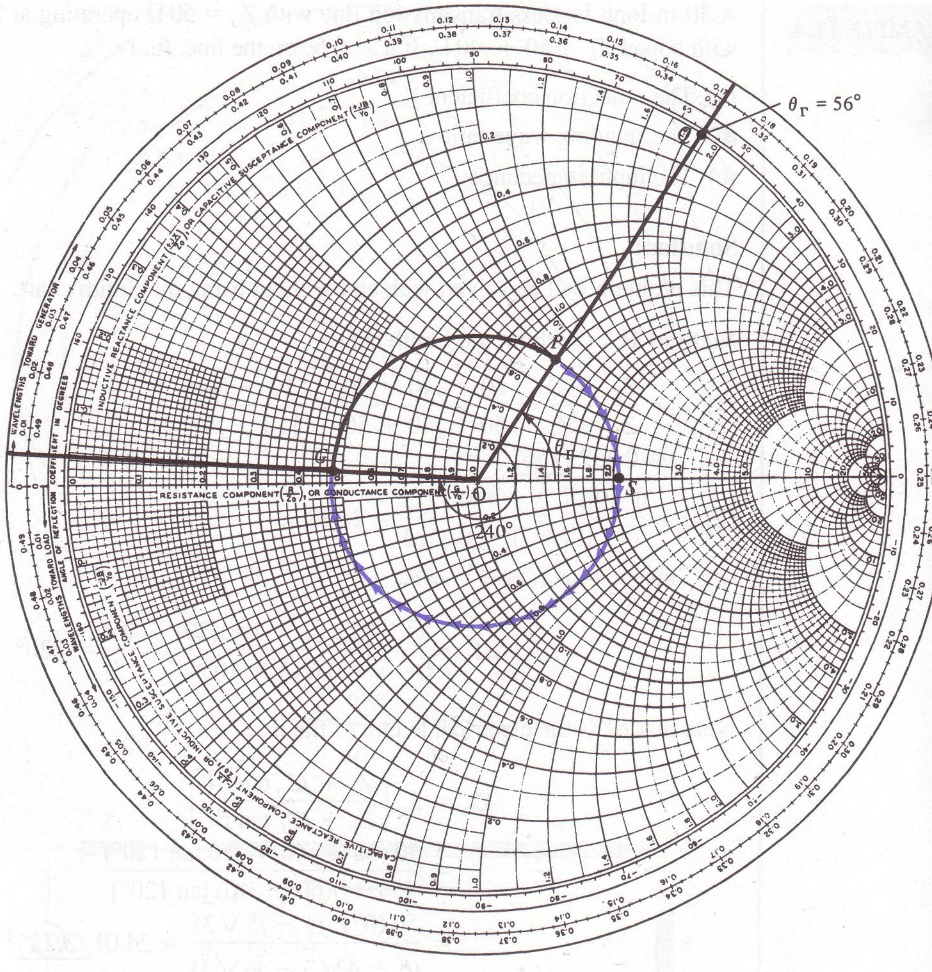


Figure 11.15 For Example 11.4.

Note that $OP = 3.2$ cm and $OQ = 9.1$ cm were taken from the Smith chart used by the author; the Smith chart in Figure 11.15 is reduced but the ratio of OP/OQ remains the same.

Angle θ_Γ is read directly on the chart as the angle between OS and OP; that is

$$\theta_\Gamma = \text{angle } POS = 56°$$

Thus

$$\Gamma = 0.3516 \underline{/56°}$$

(b) To obtain the standing wave ratio s, draw a circle with radius OP and center at O. This is the constant s or $|\Gamma|$ circle. Locate point S where the s-circle meets the Γ_r-axis.

[This is easily shown by setting $\Gamma_i = 0$ in eq. (11.49a).] The value of r at this point is s; that is

$$\begin{aligned} s &= r \text{ (for } r \geq 1) \\ &= 2.1 \end{aligned}$$

(c) To obtain Z_{in}, first express ℓ in terms of λ or in degrees.

$$\lambda = \frac{u}{f} = \frac{0.6\,(3 \times 10^8)}{2 \times 10^6} = 90 \text{ m}$$

$$\ell = 30 \text{ m} = \frac{30}{90}\lambda = \frac{\lambda}{3} \rightarrow \frac{720°}{3} = 240°$$

Since λ corresponds to an angular movement of 720° on the chart, the length of the line corresponds to an angular movement of 240°. That means we move toward the generator (or away from the load, in the clockwise direction) 240° on the s-circle from point P to point G. At G, we obtain

$$z_{in} = 0.47 + j0.035$$

Hence

$$Z_{in} = Z_o z_{in} = 50(0.47 + j0.035) = 23.5 + j1.75 \ \Omega$$

Although the results obtained using the Smith chart are only approximate, for engineering purposes they are close enough to the exact ones obtained in Method 1.

PRACTICE EXERCISE 11.4

A 70-Ω lossless line has $s = 1.6$ and $\theta_\Gamma = 300°$. If the line is 0.6λ long, obtain

(a) Γ, Z_L, Z_{in}

(b) The distance of the first minimum voltage from the load

Answer: (a) $0.228 \angle 300°$, $80.5 - j33.6 \ \Omega$, $47.6 - j17.5 \ \Omega$, (b) $\lambda/6$.

EXAMPLE 11.5

A $100 + j150$–Ω load is connected to a 75-Ω lossless line. Find:

(a) Γ

(b) s

(c) The load admittance Y_L

(d) Z_{in} at 0.4λ from the load

(e) The locations of V_{max} and V_{min} with respect to the load if the line is 0.6λ long

(f) Z_{in} at the generator.

Solution:

(a) We can use the Smith chart to solve this problem. The normalized load impedance is

$$z_L = \frac{Z_L}{Z_o} = \frac{100 + j150}{75} = 1.33 + j2$$

We locate this at point P on the Smith chart of Figure 11.16. At P, we obtain

$$|\Gamma| = \frac{OP}{OQ} = \frac{6 \text{ cm}}{9.1 \text{ cm}} = 0.659$$

$$\theta_\Gamma = \text{angle } POS = 40°$$

Hence,

$$\underline{\Gamma = 0.659 \angle 40°}$$

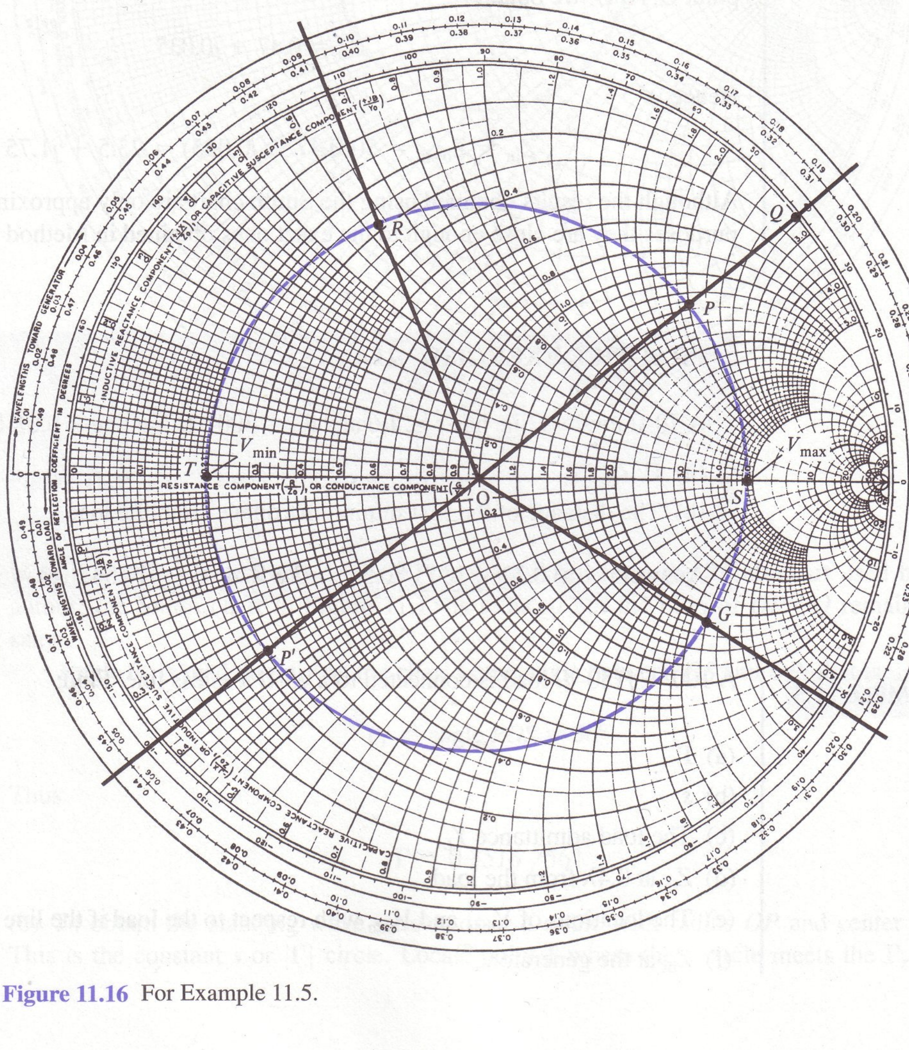


Figure 11.16 For Example 11.5.

Check:

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{100 + j150 - 75}{100 + j150 + 75}$$
$$= 0.659 \underline{/40^\circ}$$

(b) Draw the constant s-circle passing through P and obtain

$$s = 4.82$$

Check:

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.659}{1 - 0.659} = 4.865$$

(c) To obtain Y_L, extend PO to POP' and note point P' where the constant s-circle meets POP'. At P', obtain

$$y_L = 0.228 - j0.35$$

The load admittance is

$$Y_L = Y_o y_L = \frac{1}{75}(0.228 - j0.35) = 3.04 - j4.67 \text{ mS}$$

Check:

$$Y_L = \frac{1}{Z_L} = \frac{1}{100 + j150} = 3.07 - j4.62 \text{ mS}$$

(d) 0.4λ corresponds to an angular movement of $0.4 \times 720^\circ = 288^\circ$ on the constant s-circle. From P, we move 288° toward the generator (clockwise) on the s-circle to reach point R. At R,

$$z_{in} = 0.3 + j0.63$$

Hence

$$Z_{in} = Z_o z_{in} = 75\,(0.3 + j0.63)$$
$$= 22.5 + j47.25\ \Omega$$

Check:

$$\beta \ell = \frac{2\pi}{\lambda}(0.4\lambda) = 360^\circ\,(0.4) = 144^\circ$$

$$Z_{in} = Z_o \left[\frac{Z_L + jZ_o \tan \beta\ell}{Z_o + jZ_L \tan \beta\ell}\right]$$
$$= \frac{75\,(100 + j150 + j75 \tan 144^\circ)}{[75 + j(100 + j150) \tan 144^\circ]}$$
$$= 54.41 \underline{/65.25^\circ}$$

or

$$Z_{in} = 21.9 + j47.6\ \Omega$$

(e) 0.6λ corresponds to an angular movement of

$$0.6 \times 720° = 432° = 1 \text{ revolution} + 72°$$

Thus, we start from P (load end), move along the s-circle 432°, or one revolution plus 72°, and reach the generator at point G. Note that to reach G from P, we have passed through point T (location of V_{min}) once and point S (location of V_{max}) twice. Thus, from the load,

$$1\text{st } V_{max} \text{ is located at } \frac{40°}{720°}\lambda = 0.055\lambda$$

$$2\text{nd } V_{max} \text{ is located at } 0.0555\lambda + \frac{\lambda}{2} = 0.555\lambda$$

and the only V_{min} is located at $0.055\lambda + \lambda/4 = 0.3055\lambda$

(f) At G (generator end),

$$z_{in} = 1.8 - j2.2$$

$$Z_{in} = 75(1.8 - j2.2) = 135 - j165\ \Omega.$$

This can be checked by using eq. (11.34), where $\beta\ell = \dfrac{2\pi}{\lambda}(0.6\lambda) = 216°$.

We can see how much time and effort is saved using the Smith chart.

PRACTICE EXERCISE 11.5

A lossless 60-Ω line is terminated by a $60 + j60$-Ω load.

(a) Find Γ and s. If $Z_{in} = 120 - j60\ \Omega$, how far (in terms of wavelengths) is the load from the generator? Solve this without using the Smith chart.

(b) Solve the problem in (a) using the Smith chart. Calculate Z_{max} and $Z_{in,min}$. How far (in terms of λ) is the first maximum voltage from the load?

Answer: (a) $0.4472\underline{/63.43°}$, 2.618, $\dfrac{\lambda}{8}(1 + 4n)$, $n = 0, 1, 2, \ldots$, (b) $0.4457\underline{/62°}$, 2.612, $\dfrac{\lambda}{8}(1 + 4n)$, 157.1 Ω, 22.92 Ω, 0.0861 λ.

11.6 SOME APPLICATIONS OF TRANSMISSION LINES

Transmission lines are used to serve different purposes. Here we consider how transmission lines are used for load matching and impedance measurements.

A. Quarter-Wave Transformer (Matching)

When $Z_o \neq Z_L$, we say that the load is *mismatched* and a reflected wave exists on the line. However, for maximum power transfer, it is desired that the load be matched to the transmission line ($Z_o = Z_L$) so that there is no reflection ($|\Gamma| = 0$ or $s = 1$). The matching is achieved by using shorted sections of transmission lines.

We recall from eq. (11.34) that when $\ell = \lambda/4$ or $\beta\ell = (2\pi/\lambda)(\lambda/4) = \pi/2$,

$$Z_{in} = Z_o \left[\frac{Z_L + jZ_o \tan \pi/2}{Z_o + jZ_L \tan \pi/2} \right] = \frac{Z_o^2}{Z_L} \tag{11.56}$$

that is

$$\frac{Z_{in}}{Z_o} = \frac{Z_o}{Z_L}$$

or

$$z_{in} = \frac{1}{z_L} \rightarrow y_{in} = z_L \tag{11.57}$$

Thus by adding a $\lambda/4$ line on the Smith chart, we obtain the input admittance corresponding to a given load impedance.

Also, a mismatched load Z_L can be properly matched to a line (with characteristic impedance Z_o) by inserting prior to the load a transmission line $\lambda/4$ long (with characteristic impedance Z_o') as shown in Figure 11.17. The $\lambda/4$ section of the transmission line is called a *quarter-wave transformer* because it is used for impedance matching like an ordinary transformer. From eq. (11.56), Z_o' is selected such that ($Z_{in} = Z_o$)

$$Z_o' = \sqrt{Z_o Z_L} \tag{11.58}$$

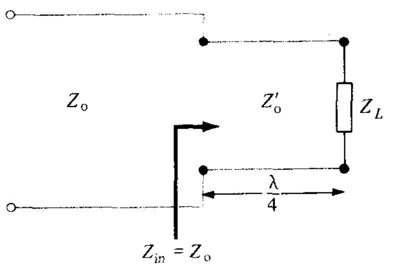


Figure 11.17 Load matching using a $\lambda/4$ transformer.

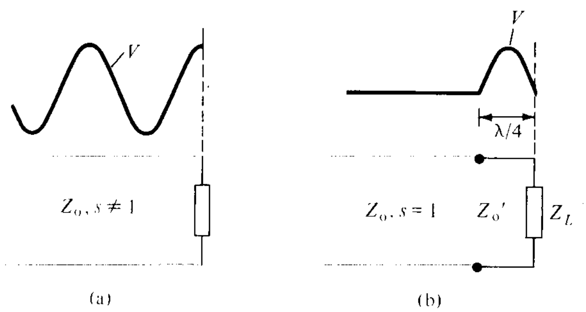


Figure 11.18 Voltage standing-wave pattern of mismatched load: **(a)** without a λ/4 transformer, **(b)** with a λ/4 transformer.

where Z_o', Z_o and Z_L are all real. If, for example, a 120-Ω load is to be matched to a 75-Ω line, the quarter-wave transformer must have a characteristic impedance of $\sqrt{(75)(120)} \simeq$ 95 Ω. This 95-Ω quarter-wave transformer will also match a 75-Ω load to a 120-Ω line. The voltage standing wave patterns with and without the λ/4 transformer are shown in Figure 11.18(a) and (b), respectively. From Figure 11.18, we observe that although a standing wave still exists between the transformer and the load, there is no standing wave to the left of the transformer due to the matching. However, the reflected wave (or standing wave) is eliminated only at the desired wavelength (or frequency f); there will be reflection at a slightly different wavelength. Thus, the main disadvantage of the quarter-wave transformer is that it is a narrow-band or frequency-sensitive device.

B. Single-Stub Tuner (Matching)

The major drawback of using a quarter-wave transformer as a line-matching device is eliminated by using a *single-stub* tuner. The tuner consists of an open or shorted section of transmission line of length d connected in parallel with the main line at some distance ℓ from the load as in Figure 11.19. Notice that the stub has the same characteristic impedance as the main line. It is more difficult to use a series stub although it is theoretically feasible. An open-circuited stub radiates some energy at high frequencies. Consequently, shunt short-circuited parallel stubs are preferred.

As we intend that $Z_{in} = Z_o$, that is, $z_{in} = 1$ or $y_{in} = 1$ at point A on the line, we first draw the locus $y = 1 + jb$ ($r = 1$ circle) on the Smith chart as shown in Figure 11.20. If a shunt stub of admittance $y_s = -jb$ is introduced at A, then

$$y_{in} = 1 + jb + y_s = 1 + jb - jb = 1 + j0 \qquad (11.59)$$

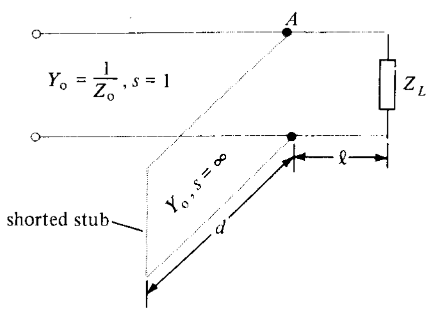


Figure 11.19 Matching with a single-stub tuner.

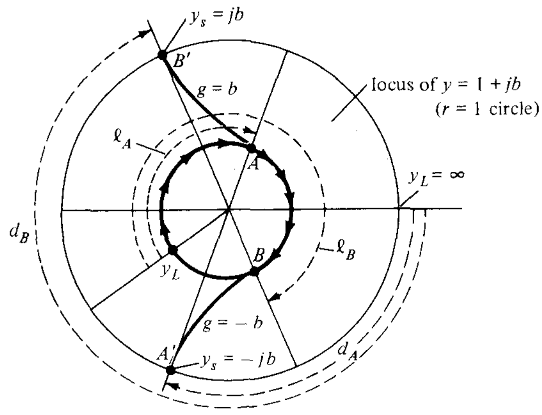


Figure 11.20 Using the Smith chart to determine ℓ and d of a shunt-shorted single-stub tuner.

as desired. Since b could be positive or negative, two possible values of ℓ ($<\lambda/2$) can be found on the line. At A, $y_s = -jb$, $\ell = \ell_A$ and at B, $y_s = jb$, $\ell = \ell_B$ as in Figure 11.20. Due to the fact that the stub is shorted ($y_L' = \infty$), we determine the length d of the stub by finding the distance from P_{sc} (at which $z_L' = 0 + j0$) to the required stub admittance y_s. For the stub at A, we obtain $d = d_A$ as the distance from P to A', where A' corresponds to $y_s = -jb$ located on the periphery of the chart as in Figure 11.20. Similarly, we obtain $d = d_B$ as the distance from P_{sc} to B' ($y_s = jb$).

Thus we obtain $d = d_A$ and $d = d_B$, corresponding to A and B, respectively, as shown in Figure 11.20. Note that $d_A + d_B = \lambda/2$ always. Since we have two possible shunted stubs, we normally choose to match the shorter stub or one at a position closer to the load. Instead of having a single stub shunted across the line, we may have two stubs. This is called *double-stub matching* and allows for the adjustment of the load impedance.

C. Slotted Line (Impedance Measurement)

At high frequencies, it is very difficult to measure current and voltage because measuring devices become significant in size and every circuit becomes a transmission line. The slotted line is a simple device used in determining the impedance of an unknown load at high frequencies up into the region of gigahertz. It consists of a section of an air (lossless) line with a slot in the outer conductor as shown in Figure 11.21. The line has a probe, along the **E** field (see Figure 11.4), which samples the **E** field and consequently measures the potential difference between the probe and its outer shield.

The slotted line is primarily used in conjunction with the Smith chart to determine the standing wave ratio s (the ratio of maximum voltage to the minimum voltage) and the load impedance Z_L. The value of s is read directly on the detection meter when the load is connected. To determine Z_L, we first replace the load by a short circuit and note the locations of voltage minima (which are more accurately determined than the maxima because of the sharpness of the turning point) on the scale. Since impedances repeat every half wavelength, any of the minima may be selected as the load reference point. We now determine the distance from the selected reference point to the load by replacing the short circuit by the load and noting the locations of voltage minima. The distance ℓ (dis-

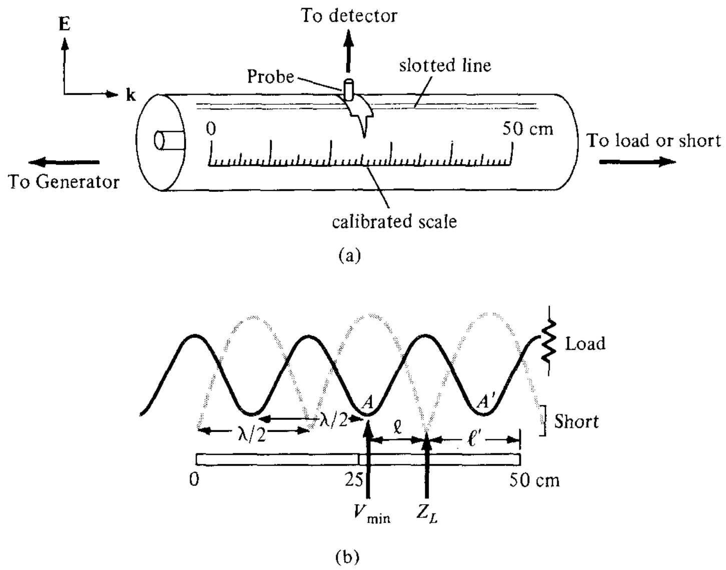


Figure 11.21 **(a)** Typical slotted line; **(b)** determining the location of the load Z_L and V_{min} on the line.

tance of V_{min} toward the load) expressed in terms of λ is used to locate the position of the load of an s-circle on the chart as shown in Figure 11.22. We could also locate the load by using ℓ', which is the distance of V_{min} toward the generator. Either ℓ or ℓ' may be used to locate Z_L.

The procedure involved in using the slotted line can be summarized as follows:

1. With the load connected, read s on the detection meter. With the value of s, draw the s-circle on the Smith chart.
2. With the load replaced by a short circuit, locate a reference position for z_L at a voltage minimum point.
3. With the load on the line, note the position of V_{min} and determine ℓ.
4. On the Smith chart, move toward the load a distance ℓ from the location of V_{min}. Find Z_L at that point.

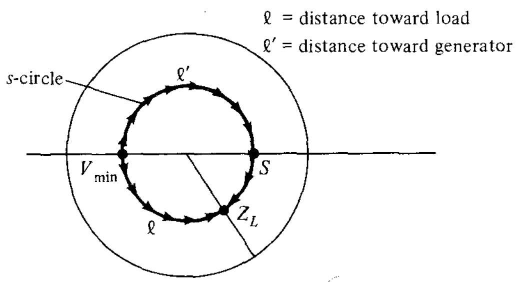


Figure 11.22 Determining the load impedance from the Smith chart using the data obtained from the slotted line.

EXAMPLE 11.6

With an unknown load connected to a slotted air line, $s = 2$ is recorded by a standing wave indicator and minima are found at 11 cm, 19 cm, . . . on the scale. When the load is replaced by a short circuit, the minima are at 16 cm, 24 cm, ... If $Z_o = 50\ \Omega$, calculate λ, f, and Z_L.

Solution:

Consider the standing wave patterns as in Figure 11.23(a). From this, we observe that

$$\frac{\lambda}{2} = 19 - 11 = 8\text{ cm} \qquad \text{or} \qquad \lambda = 16\text{ cm}$$

$$f = \frac{u}{\lambda} = \frac{3 \times 10^8}{16 \times 10^{-2}} = 1.875\text{ GHz}$$

Electrically speaking, the load can be located at 16 cm or 24 cm. If we assume that the load is at 24 cm, the load is at a distance ℓ from V_{min}, where

$$\ell = 24 - 19 = 5\text{ cm} = \frac{5}{16}\lambda = 0.3125\,\lambda$$

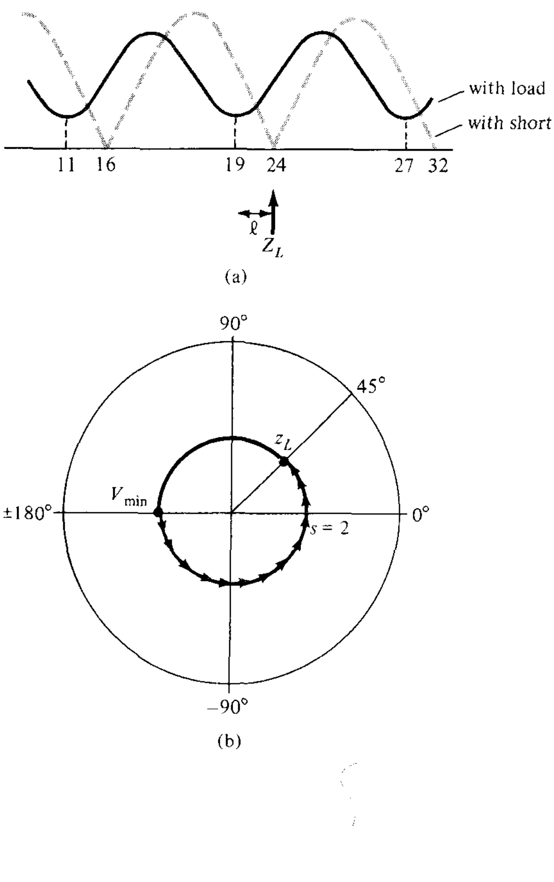


Figure 11.23 Determining Z_L using the slotted line: **(a)** wave pattern, **(b)** Smith chart for Example 11.6.

This corresponds to an angular movement of $0.3125 \times 720° = 225°$ on the $s = 2$ circle. By starting at the location of V_{min} and moving 225° toward the load (counterclockwise), we reach the location of z_L as illustrated in Figure 11.23(b). Thus

$$z_L = 1.4 + j0.75$$

and

$$Z_L = Z_o z_L = 50\,(1.4 + j0.75) = 70 + j37.5\ \Omega$$

PRACTICE EXERCISE 11.6

The following measurements were taken using the slotted line technique: with load, $s = 1.8$, V_{max} occurred at 23 cm, 33.5 cm, . . .; with short, $s = \infty$, V_{max} occurred at 25 cm, 37.5 cm, ... If $Z_o = 50\ \Omega$, determine Z_L.

Answer: $32.5 - j17.5\ \Omega$.

EXAMPLE 11.7

Antenna with impedance $40 + j30\ \Omega$ is to be matched to a 100-Ω lossless line with a shorted stub. Determine

(a) The required stub admittance
(b) The distance between the stub and the antenna
(c) The stub length
(d) The standing wave ratio on each ratio of the system

Solution:

(a) $z_L = \dfrac{Z_L}{Z_o} = \dfrac{40 + j30}{100} = 0.4 + j0.3$

Locate z_L on the Smith chart as in Figure 11.24 and from this draw the s-circle so that y_L can be located diametrically opposite z_L. Thus $y_L = 1.6 - j1.2$. Alternatively, we may find y_L using

$$y_L = \frac{Z_o}{Z_L} = \frac{100}{40 + j30} = 1.6 - j1.2$$

Locate points A and B where the s-circle intersects the $g = 1$ circle. At A, $y_s = -j1.04$ and at B, $y_s = +j1.04$. Thus the required stub admittance is

$$Y_s = Y_o y_s = \pm j1.04 \frac{1}{100} = \pm j10.4\ \text{m}S$$

Both $j10.4$ mS and $-j10.4$ mS are possible values.

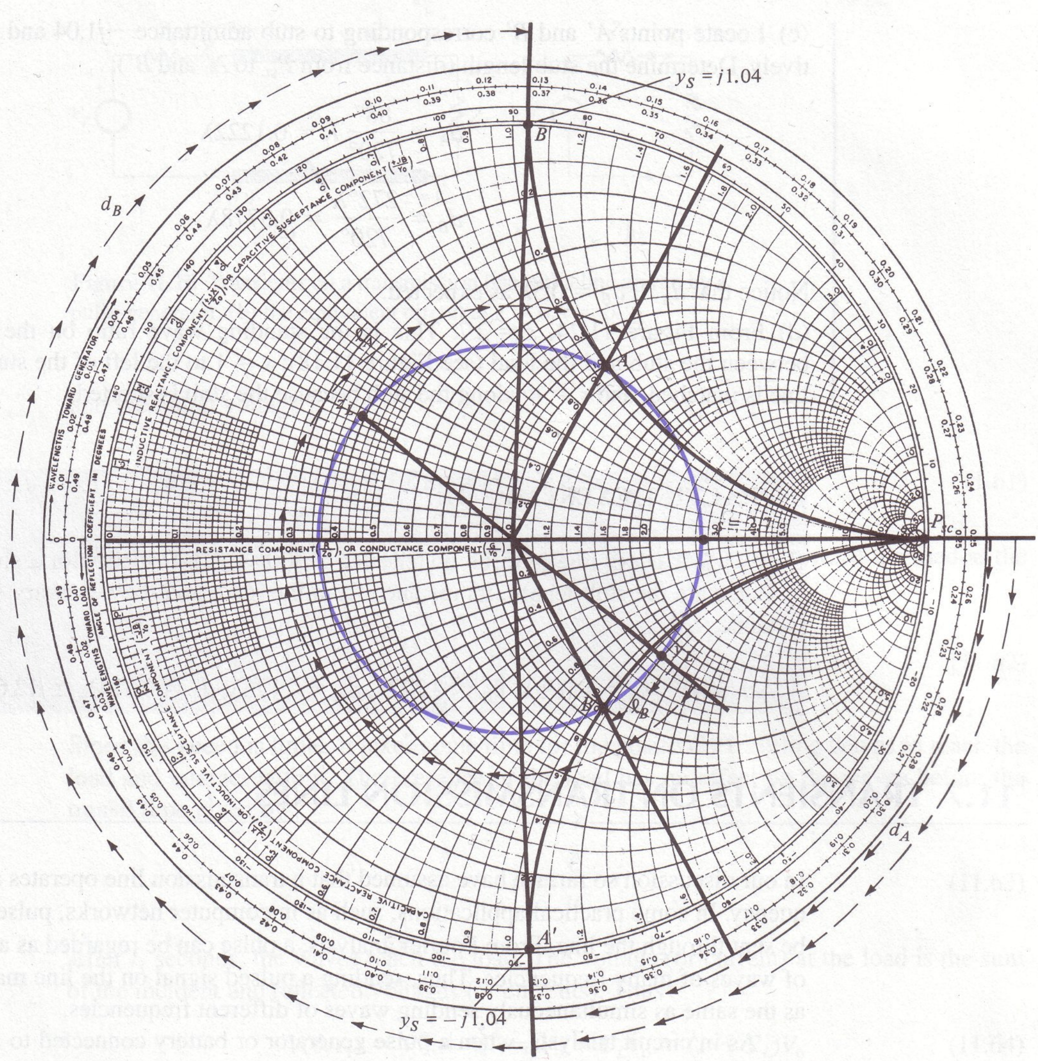


Figure 11.24 For Example 11.7.

(b) From Figure 11.24, we determine the distance between the load (antenna in this case) y_L and the stub. At A,

$$\ell_A = \frac{\lambda}{2} - \frac{(62° - -39°)\lambda}{720°} = 0.36\lambda$$

At B:

$$\ell_B = \frac{(62° - 39°)}{720°} = 0.032\lambda$$

(c) Locate points A' and B' corresponding to stub admittance $-j1.04$ and $j1.04$, respectively. Determine the stub length (distance from P_{sc} to A' and B'):

$$d_A = \frac{88°}{720°}\lambda = 0.1222\lambda$$

$$d_B = \frac{272°\lambda}{720°} = 0.3778\lambda$$

Notice that $d_A + d_B = 0.5\lambda$ as expected.

(d) From Figure 11.24, $s = 2.7$. This is the standing wave ratio on the line segment between the stub and the load (see Figure 11.18); $s = 1$ to the left of the stub because the line is matched, and $s = \infty$ along the stub because the stub is shorted.

PRACTICE EXERCISE 11.7

A 75-Ω lossless line is to be matched to a $100 - j80$–Ω load with a shorted stub. Calculate the stub length, its distance from the load, and the necessary stub admittance.

Answer: $\ell_A = 0.093\lambda$, $\ell_B = 0.272\lambda$, $d_A = 0.126\lambda$, $d_B = 0.374\lambda$, $\pm j12.67$ mS.

11.7 TRANSIENTS ON TRANSMISSION LINES

In our discussion so far, we have assumed that a transmission line operates at a single frequency. In some practical applications, such as in computer networks, pulsed signals may be sent through the line. From Fourier analysis, a pulse can be regarded as a superposition of waves of many frequencies. Thus, sending a pulsed signal on the line may be regarded as the same as simultaneously sending waves of different frequencies.

As in circuit analysis, when a pulse generator or battery connected to a transmission line is switched on, it takes some time for the current and voltage on the line to reach steady values. This transitional period is called the *transient.* The transient behavior just after closing the switch (or due to lightning strokes) is usually analyzed in the frequency domain using Laplace transform. For the sake of convenience, we treat the problem in the time domain.

Consider a lossless line of length ℓ and characteristic impedance Z_o as shown in Figure 11.25(a). Suppose that the line is driven by a pulse generator of voltage V_g with internal impedance Z_g at $z = 0$ and terminated with a purely resistive load Z_L. At the instant $t = 0$ that the switch is closed, the starting current "sees" only Z_g and Z_o, so the initial situation can be described by the equivalent circuit of Figure 11.25(b). From the figure, the starting current at $z = 0$, $t = 0^+$ is given by

$$I(0, 0^+) = I_o = \frac{V_g}{Z_g + Z_o} \tag{11.60}$$

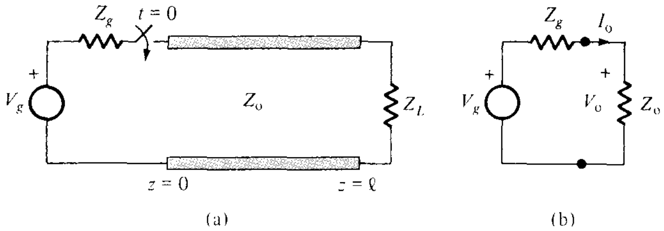


Figure 11.25 Transients on a transmission line: **(a)** a line driven by a pulse generator, **(b)** the equivalent circuit at $z = 0, t = 0^+$.

and the initial voltage is

$$V(0, 0^+) = V_o = I_o Z_o = \frac{Z_o}{Z_g + Z_o} V_g \tag{11.61}$$

After the switch is closed, waves $I^+ = I_o$ and $V^+ = V_o$ propagate toward the load at the speed

$$u = \frac{1}{\sqrt{LC}} \tag{11.62}$$

Since this speed is finite, it takes some time for the positively traveling waves to reach the load and interact with it. The presence of the load has no effect on the waves before the transit time given by

$$t_1 = \frac{\ell}{u} \tag{11.63}$$

After t_1 seconds, the waves reach the load. The voltage (or current) at the load is the sum of the incident and reflected voltages (or currents). Thus

$$V(\ell, t_1) = V^+ + V^- = V_o + \Gamma_L V_o = (1 + \Gamma_L)V_o \tag{11.64}$$

and

$$I(\ell, t_1) = I^+ + I^- = I_o - \Gamma_L I_o = (1 - \Gamma_L)I_o \tag{11.65}$$

where Γ_L is the load reflection coefficient given in eq. (11.36); that is,

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} \tag{11.66}$$

The reflected waves $V^- = \Gamma_L V_o$ and $I^- = -\Gamma_L I_o$ travel back toward the generator in addition to the waves V_o and I_o already on the line. At time $t = 2t_1$, the reflected waves have reached the generator, so

$$V(0, 2t_1) = V^+ + V^- = \Gamma_G \Gamma_L V_o + (1 + \Gamma_L)V_o$$

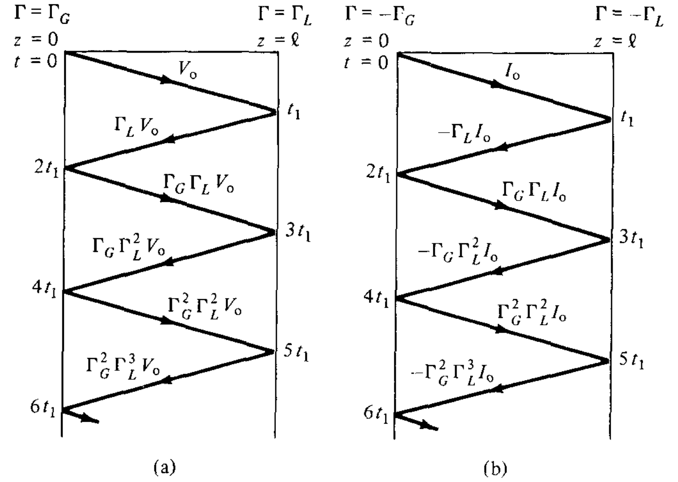


Figure 11.26 Bounce diagram for **(a)** a voltage wave, and **(b)** a current wave.

or

$$V(0, 2t_1) = (1 + \Gamma_L + \Gamma_G\Gamma_L)V_o \tag{11.67}$$

and

$$I(0, 2t_1) = I^+ + I^- = -\Gamma_G(-\Gamma_L I_o) + (1 - \Gamma_L)I_o$$

or

$$I(0, 2t_1) = (1 - \Gamma_L + \Gamma_L\Gamma_G)I_o \tag{11.68}$$

where Γ_G is the generator reflection coefficient given by

$$\Gamma_G = \frac{Z_g - Z_o}{Z_g + Z_o} \tag{11.69}$$

Again the reflected waves (from the generator end) $V^+ = \Gamma_G\Gamma_L V_o$ and $I^+ = \Gamma_G\Gamma_L I_o$ propagate toward the load and the process continues until the energy of the pulse is actually absorbed by the resistors Z_g and Z_L.

Instead of tracing the voltage and current waves back and forth, it is easier to keep track of the reflections using a *bounce diagram,* otherwise known as a *lattice diagram.* The bounce diagram consists of a zigzag line indicating the position of the voltage (or current) wave with respect to the generator end as shown in Figure 11.26. On the bounce diagram, the voltage (or current) at any time may be determined by adding those values that appear on the diagram above that time.

EXAMPLE 11.8

For the transmission line of Figure 11.27, calculate and sketch

(a) The voltage at the load and generator ends for $0 < t < 6\ \mu s$

(b) The current at the load and generator ends for $0 < t < 6\ \mu s$

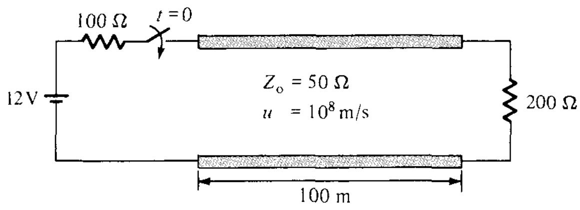


Figure 11.27 For Example 11.8.

Solution:

(a) We first calculate the voltage reflection coefficients at the generator and load ends.

$$\Gamma_G = \frac{Z_g - Z_o}{Z_g + Z_o} = \frac{100 - 50}{100 + 50} = \frac{1}{3}$$

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{200 - 50}{200 + 50} = \frac{3}{5}$$

The transit time $t_1 = \frac{\ell}{u} = \frac{100}{10^8} = 1\ \mu\text{s}$.

The initial voltage at the generator end is

$$V_o = \frac{Z_o}{Z_o + Z_g} V_g = \frac{50}{150}(12) = 4\ \text{V}$$

The 4 V is sent out to the load. The leading edge of the pulse arrives at the load at $t = t_1 = 1\ \mu s$. A portion of it, 4(3/5) = 2.4 V, is reflected back and reaches the generator at $t = 2t_1 = 2\ \mu\text{s}$. At the generator, 2.4(1/3) = 0.8 is reflected and the process continues. The whole process is best illustrated in the voltage bounce diagram of Figure 11.28.

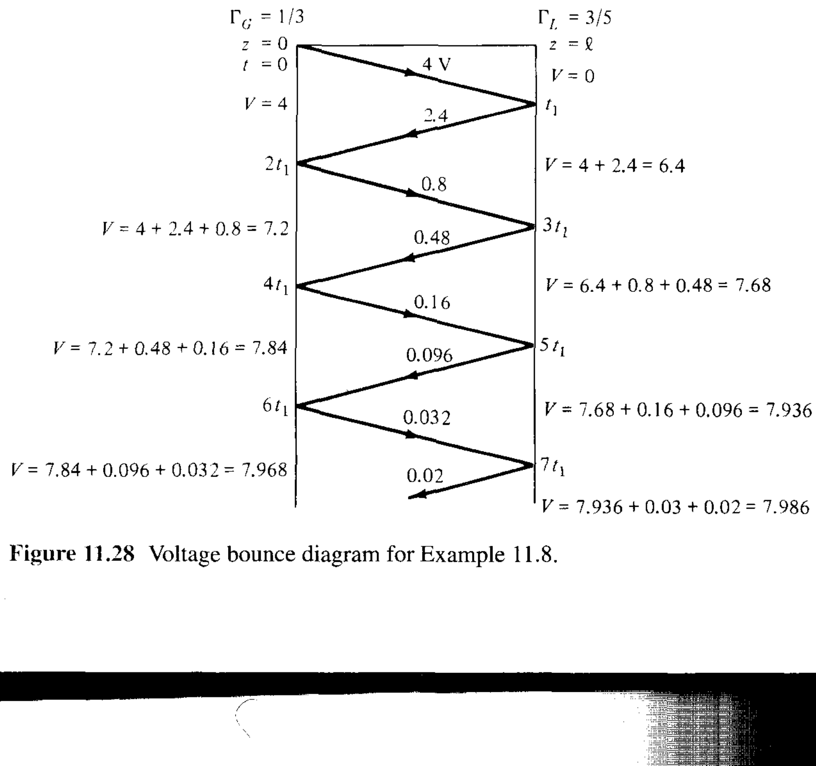


Figure 11.28 Voltage bounce diagram for Example 11.8.

From the bounce diagram, we can sketch $V(0, t)$ and $V(\ell, t)$ as functions of time as shown in Figure 11.29. Notice from Figure 11.29 that as $t \to \infty$, the voltages approach an asymptotic value of

$$V_\infty = \frac{Z_L}{Z_L + Z_g} V_g = \frac{200}{300}(12) = 8 \text{ V}$$

This should be expected because the equivalent circuits at $t = 0$ and $t = \infty$ are as shown in Figure 11.30 (see Problem 11.46 for proof).

(b) The current reflection coefficients at the generator and load ends are $-\Gamma_G = -1/3$ and $-\Gamma_L = -3/5$, respectively. The initial current is

$$I_o = \frac{V_o}{Z_o} = \frac{4}{50} = 80 \text{ mA}$$

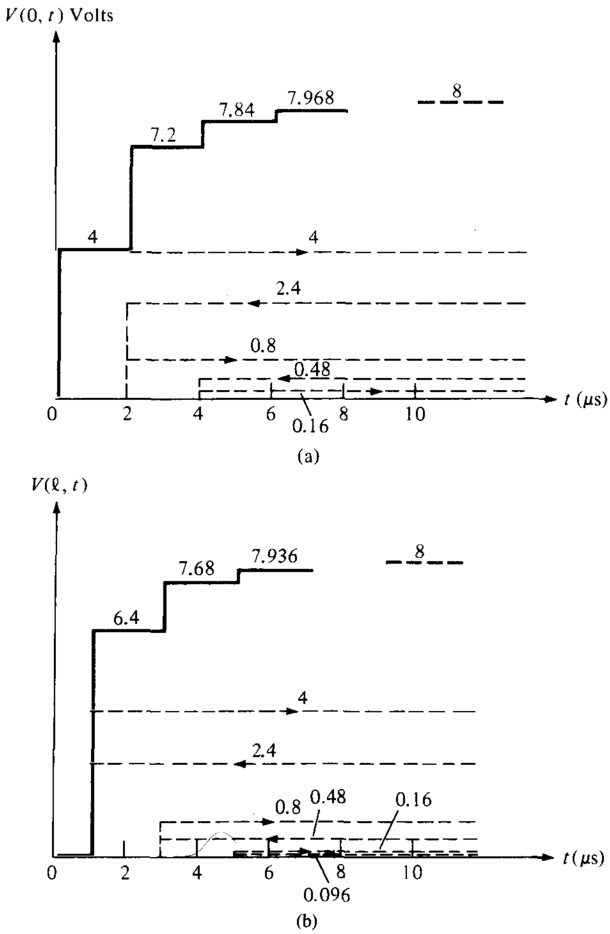


Figure 11.29 Voltage (not to scale): **(a)** at the generator end, **(b)** at the load end.

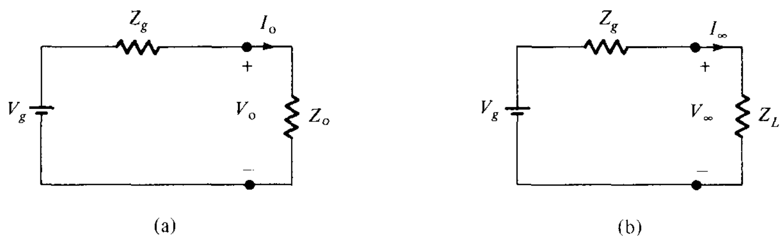


Figure 11.30 Equivalent circuits for the line in Figure 11.27 for **(a)** $t = 0$, and **(b)** $t = \infty$.

Again, $I(0, t)$ and $I(\ell, t)$ are easily obtained from the current bounce diagram shown in Figure 11.31. These currents are sketched in Figure 11.32. Note that $I(\ell, t) = V(\ell, t)/Z_L$. Hence, Figure 11.32(b) can be obtained either from the current bounce diagram of Figure 11.31 or by scaling Figure 11.29(b) by a factor of $1/Z_L = 1/200$. Notice from Figures 11.30(b) and 11.32 that the currents approach an asymptotic value of

$$I_\infty = \frac{V_g}{Z_g + Z_L} = \frac{12}{300} = 40 \text{ mA}$$

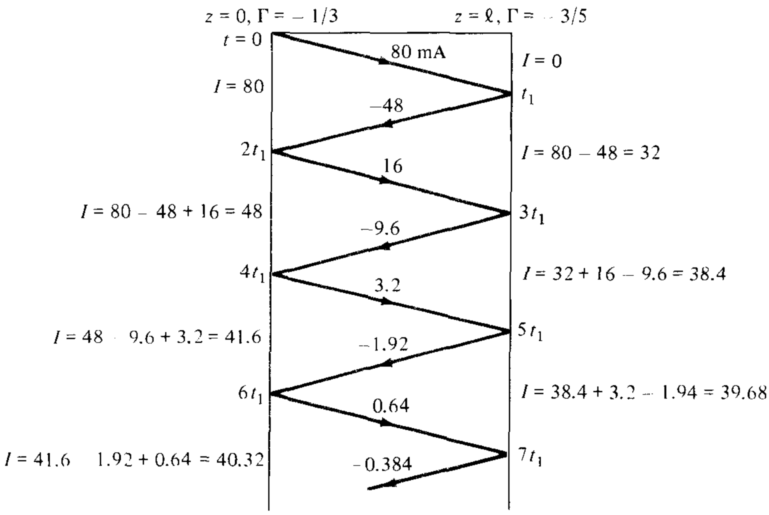


Figure 11.31 Current bounce diagram for Example 11.8.

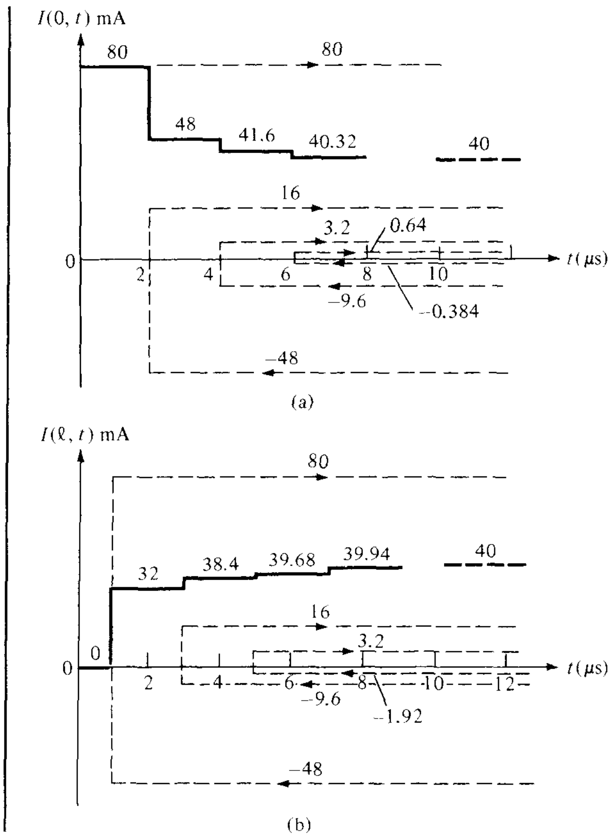


Figure 11.32 Current (not to scale): **(a)** at the generator end, **(b)** at the load end, for Example 11.8.

PRACTICE EXERCISE 11.8

Repeat Example 11.8 if the transmission line is

(a) Short-circuited

(b) Open-circuited

Answer: (a) See Figure 11.33
(b) See Figure 11.34

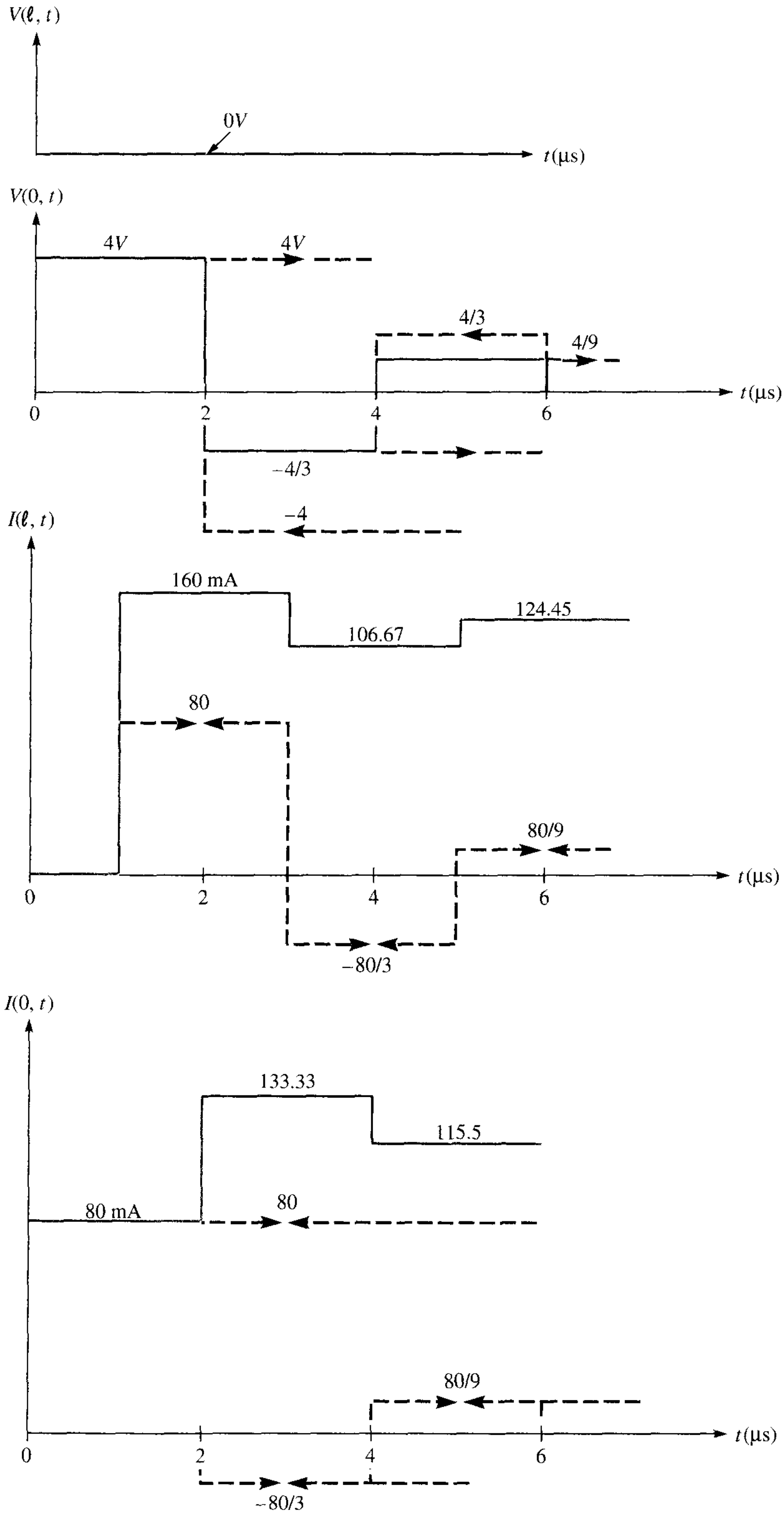


Figure 11.33 For Practice Exercise 11.8(a).

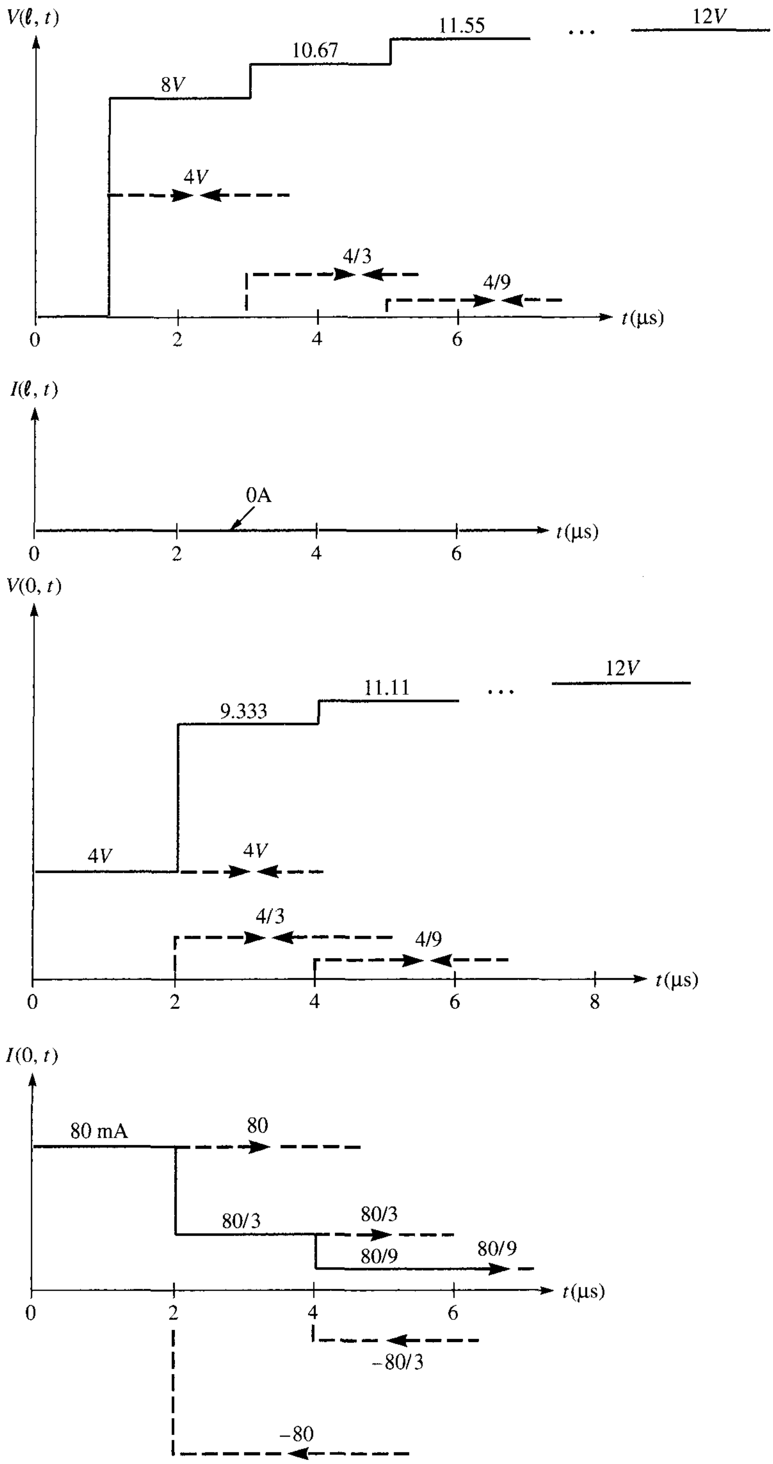


Figure 11.34 For Practice Exercise 11.8(b).

EXAMPLE 11.9

A 75-Ω transmission line of length 60 m is terminated by a 100-Ω load. If a rectangular pulse of width 5 μs and magnitude 4 V is sent out by the generator connected to the line, sketch $I(0, t)$ and $I(\ell, t)$ for $0 < t < 15\ \mu$s. Take $Z_g = 25\ \Omega$ and $u = 0.1c$.

Solution:

In the previous example, the switching on of a battery created a step function, a pulse of infinite width. In this example, the pulse is of finite width of 5 μs. We first calculate the voltage reflection coefficients:

$$\Gamma_G = \frac{Z_g - Z_o}{Z_g + Z_o} = -\frac{1}{2}$$

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{1}{7}$$

The initial voltage and transit time are given by

$$V_o = \frac{Z_o}{Z_o + Z_g} V_g = \frac{75}{100}(4) = 3\ V$$

$$t_1 = \frac{\ell}{u} = \frac{60}{0.1\,(3 \times 10^8)} = 2\ \mu s$$

The time taken by V_o to go forth and back is $2t_1 = 4\ \mu$s, which is less than the pulse duration of 5 μs. Hence, there will be overlapping.

The current reflection coefficients are

$$-\Gamma_L = -\frac{1}{7} \quad \text{and} \quad -\Gamma_G = \frac{1}{2}$$

The initial current $I_o = \dfrac{V_g}{Z_g + Z_o} = \dfrac{4}{100} = 40$ mA.

Let i and r denote incident and reflected pulses, respectively. At the generator end:

$$0 < t < 5\ \mu\text{s}, \qquad I_r = I_o = 40\ \text{mA}$$

$$4 < t < 9, \qquad I_i = -\frac{1}{7}(40) = -5.714$$

$$I_r = \frac{1}{2}(-5.714) = -2.857$$

$$8 < t < 13, \qquad I_i = -\frac{1}{7}(-2.857) = 0.4082$$

$$I_r = \frac{1}{2}(0.4082) = 0.2041$$

$$12 < t < 17, \quad I_i = -\frac{1}{7}(0.2041) = -0.0292$$

$$I_r = \frac{1}{2}(-0.0292) = -0.0146$$

and so on. Hence, the plot of $I(0, t)$ versus t is as shown in Figure 11.35(a).

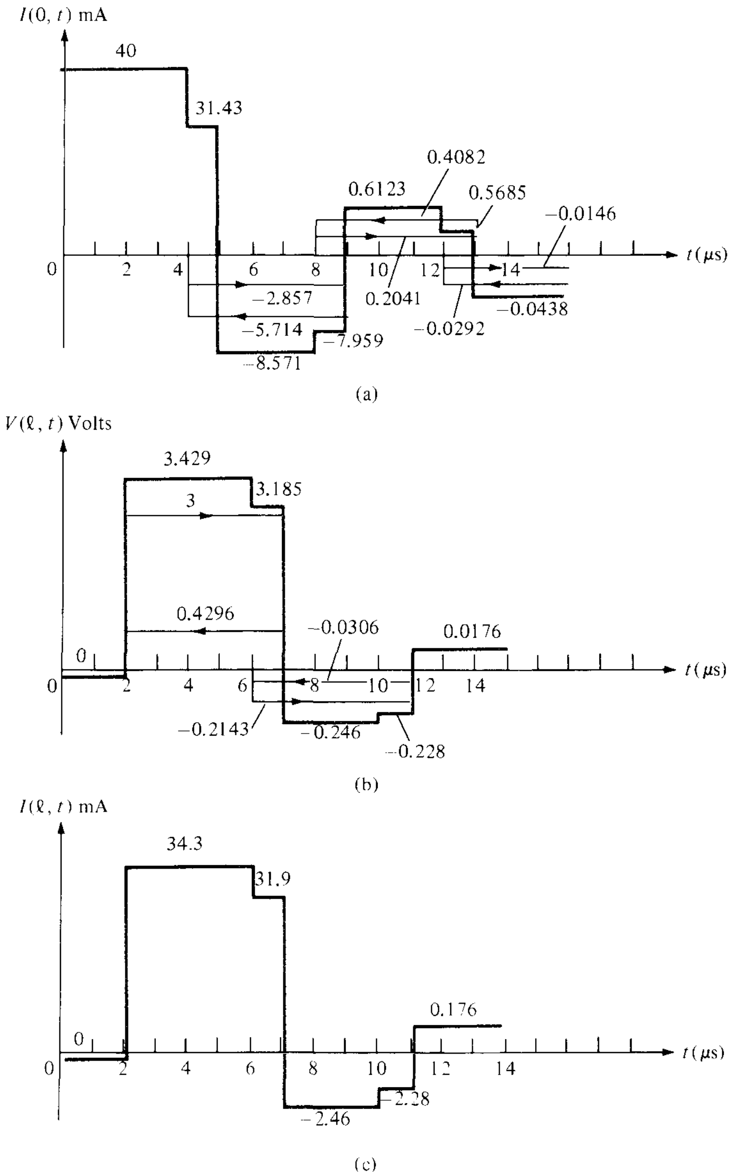


Figure 11.35 For Example 11.9 (not to scale).

At the load end:

$$0 < t < 2\ \mu s, \qquad V = 0$$

$$2 < t < 7, \qquad V_i = 3$$

$$V_r = \frac{1}{7}(3) = 0.4296$$

$$6 < t < 11, \qquad V_i = -\frac{1}{2}(0.4296) = -0.2143$$

$$V_r = \frac{1}{7}(-0.2143) = -0.0306$$

$$10 < t < 14, \qquad V_i = -\frac{1}{2}(-0.0306) = 0.0154$$

$$V_r = \frac{1}{7}(0.0154) = 0.0022$$

and so on. From $V(\ell, t)$, we can obtain $I(\ell, t)$ as

$$I(\ell, t) = \frac{V(\ell, t)}{Z_o} = \frac{V(\ell, t)}{100}$$

The plots of $V(\ell, t)$ and $I(\ell, t)$ are shown in Figure 11.35(b) and (c).

PRACTICE EXERCISE 11.9

Repeat Example 11.9 if the rectangular pulse is replaced by a triangular pulse of Figure 11.36.

Answer: $(I_o)_{max} = 100$ mA. See Figure 11.37 for the current waveforms.

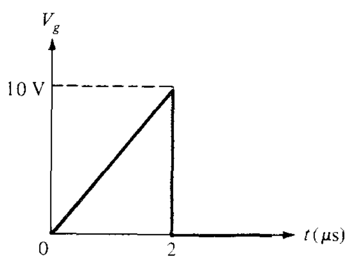


Figure 11.36 Triangular pulse of Practice Exercise 11.9.

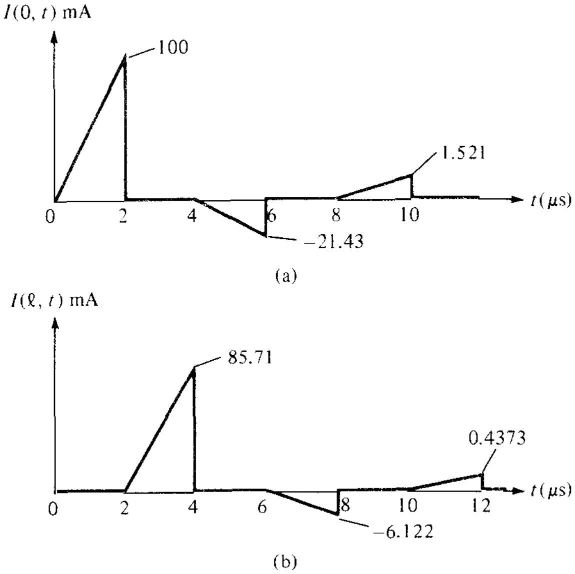


Figure 11.37 Current waves for Practice Exercise 11.9.

11.8 MICROSTRIP TRANSMISSION LINES

Microstrip lines belong to a group of lines known as parallel-plate transmission lines. They are widely used in present-day electronics. Apart from being the most commonly used form of transmission lines for microwave integrated circuits, microstrips are used for circuit components such as filters, couplers, resonators, antennas, and so on. In comparison with the coaxial line, the microstrip line allows for greater flexibility and compactness of design.

A microstrip line consists of a single ground plane and an open strip conductor separated by dielectric substrate as shown in Figure 11.38. It is constructed by the photographic processes used for integrated circuits. Analytical derivation of the characteristic properties

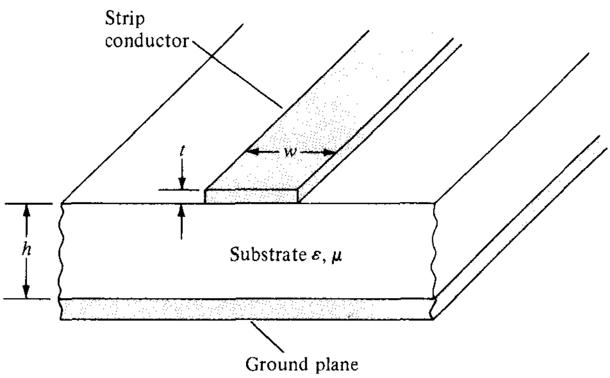


Figure 11.38 Microstrip line.

of the line is cumbersome. We will consider only some basic, valid empirical formulas necessary for calculating the phase velocity, impedance, and losses of the line.

Due to the open structure of the microstrip line, the EM field is not confined to the dielectric, but is partly in the surrounding air as in Figure 11.39. Provided the frequency is not too high, the microstrip line will propagate a wave that, for all practical purposes, is a TEM wave. Because of the fringing, the *effective relative permittivity* ε_{eff} is less than the relative permittivity ε_r of the substrate. If w is the line width and h is the substrate thickness, an a approximate value of ε_{eff} is given by

$$\boxed{\varepsilon_{\text{eff}} = \frac{(\varepsilon_r + 1)}{2} + \frac{(\varepsilon_r - 1)}{2\sqrt{1 + 12h/w}}} \tag{11.70}$$

The characteristic impedance is given by the following approximate formulas:

$$\boxed{Z_o = \begin{cases} \dfrac{60}{\sqrt{\varepsilon_{\text{eff}}}} \ln\left(\dfrac{8h}{w} + \dfrac{w}{h}\right), & w/h \leq 1 \\[2ex] \dfrac{1}{\sqrt{\varepsilon_{\text{eff}}}} \dfrac{120\pi}{[w/h + 1.393 + 0.667 \ln(w/h + 1.444)]}, & w/h \geq 1 \end{cases}} \tag{11.71}$$

The characteristic impedance of a wide strip is often low while that of a narrow strip is high.

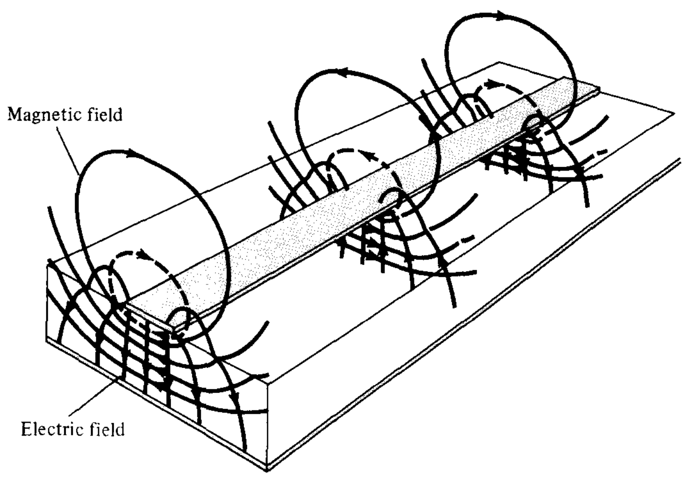


Figure 11.39 Pattern of the EM field of a microstrip line. *Source:* From D. Roddy, *Microwave Technology,* 1986, by permission of Prentice-Hall.

For design purposes, if ε_r and Z_o are known, the ratio w/h necessary to achieve Z_o is given by

$$\frac{w}{h} = \begin{cases} \dfrac{8e^A}{e^{2A} - 2}, & w/h < 2 \\ \dfrac{2}{\pi}\left\{B - 1 - \ln(2B - 1)\right. \\ \left. + \dfrac{\varepsilon_r - 1}{2\varepsilon_r}\left[\ln(B - 1) + 0.39 - \dfrac{0.61}{\varepsilon_r}\right]\right\}, & w/h > 2 \end{cases} \tag{11.72}$$

where

$$A = \frac{Z_o}{60}\sqrt{\frac{\varepsilon_r - 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right)$$

$$B = \frac{60\pi^2}{Z_o\sqrt{\varepsilon_r}}$$

From the knowledge of ε_{eff} and Z_o, the phase constant and the phase velocity of a wave propagating on the microstrip are given by

$$\beta = \frac{\omega\varepsilon_{\text{eff}}}{c} \tag{11.74a}$$

$$u = \frac{c}{\varepsilon_{\text{eff}}} \tag{11.74b}$$

where c is the speed of light in a vacuum. The attenuation due to conduction (or ohmic) loss is (in dB/m)

$$\alpha_c \simeq 8.686\frac{R_s}{wZ_o} \tag{11.75}$$

where $R_s = \dfrac{1}{\sigma_c\delta}$ is the skin resistance of the conductor. The attenuation due to dielectric loss is (in dB/m)

$$\alpha_d \simeq 27.3\frac{(\varepsilon_{\text{eff}} - 1)\,\varepsilon_r}{(\varepsilon_r - 1)\,\varepsilon_{\text{eff}}}\frac{\tan\theta}{\lambda} \tag{11.76}$$

where $\lambda = u/f$ is the line wavelength and $\tan\theta = \sigma/\omega\varepsilon$ is the loss tangent of the substrate. The total attenuation constant is the sum of the ohmic attenuation constant α_c and the dielectric attenuation constant α_d, that is,

$$\alpha = \alpha_c + \alpha_d \tag{11.77}$$

Sometimes α_d is negligible in comparison with α_c. Although they offer an advantage of flexibility and compactness, the microstrip lines are not useful for long transmission due to excessive attenuation.

EXAMPLE 11.10

A certain microstrip line has fused quartz ($\varepsilon_r = 3.8$) as a substrate. If the ratio of line width to substrate thickness is $w/h = 4.5$, determine

(a) The effective relative permittivity of the substrate

(b) The characteristic impedance of the line

(c) The wavelength of the line at 10 GHz

Solution:

(a) For $w/h = 4.5$, we have a wide strip. From eq. (11.70),

$$\varepsilon_{\text{eff}} = \frac{4.8}{2} + \frac{2.8}{2}\left[1 + \frac{12}{4.5}\right]^{-1/2} = 3.131$$

(b) From eq. (11.71),

$$\begin{aligned} Z_o &= \frac{120\pi}{\sqrt{3.131}[4.5 + 1.393 + 0.667 \ln (4.5 + 1.444)]} \\ &= 9.576\ \Omega \end{aligned}$$

(c) $\lambda = \dfrac{u}{f} = \dfrac{c}{f\sqrt{\varepsilon_{\text{eff}}}} = \dfrac{3 \times 10^8}{10^{10}\sqrt{3.131}}$
$= 1.69 \times 10^{-2}\text{ m} = 16.9\text{ mm}$

PRACTICE EXERCISE 11.10

Repeat Example 11.10 for $w/h = 0.8$.

Answer: (a) 2.75, (b) 84.03 Ω, (c) 18.09 mm.

EXAMPLE 11.11

At 10 GHz, a microstrip line has the following parameters:

$$h = 1\text{ mm}$$

$$w = 0.8\text{ mm}$$

$$\varepsilon_r = 6.6$$

$$\tan\theta = 10^{-4}$$

$$\sigma_c = 5.8 \times 10^7\text{ S/m}$$

Calculate the attenuation due to conduction loss and dielectric loss.

Solution:

The ratio $w/h = 0.8$. Hence from eqs. (11.70) and (11.71)

$$\varepsilon_e = \frac{7.2}{2} + \frac{5.6}{2}\left(1 + \frac{12}{0.8}\right)^{-1/2} = 4.3$$

$$Z_o = \frac{60}{\sqrt{4.3}} \ln\left(\frac{8}{0.8} + \frac{0.8}{4}\right)$$
$$= 67.17\ \Omega$$

The skin resistance of the conductor is

$$R_s = \frac{1}{\sigma_c \delta} = \sqrt{\frac{\pi f \mu_o}{\sigma_c}} = \sqrt{\frac{\pi \times 10 \times 10^9 \times 4\pi \times 10^{-7}}{5.8 \times 10^7}}$$
$$= 2.609 \times 10^{-2}\ \Omega/\text{m}^2$$

Using eq. (11.75), we obtain the conduction attenuation constant as

$$\alpha_c = 8.686 \times \frac{2.609 \times 10^{-2}}{0.8 \times 10^{-3} \times 67.17}$$
$$= 4.217\ \text{dB/m}$$

To find the dielectric attenuation constant, we need λ.

$$\lambda = \frac{u}{f} = \frac{c}{f\sqrt{\varepsilon_{\text{eff}}}} = \frac{3 \times 10^8}{10 \times 10^9 \sqrt{4.3}}$$
$$= 1.447 \times 10^{-2}\ \text{m}$$

Applying eq. (11.76), we have

$$\alpha_d = 27.3 \times \frac{3.492 \times 6.6 \times 10^{-4}}{5.6 \times 4.3 \times 1.447 \times 10^{-2}}$$
$$= 0.1706\ \text{dB/m}$$

PRACTICE EXERCISE 11.11

Calculate the attenuation due to ohmic losses at 20 GHz for a microstrip line constructed of copper conductor having a width of 2.5 mm on an alumina substrate. Take the characteristic impedance of the line as 50 Ω.

Answer: 2.564 dB/m.

SUMMARY

1. A transmission line is commonly described by its distributed parameters R (in Ω/m), L (in H/m), G (in S/m), and C (in F/m). Formulas for calculating R, L, G, and C are provided in Table 11.1 for coaxial, two-wire, and planar lines.

2. The distributed parameters are used in an equivalent circuit model to represent a differential length of the line. The transmission-line equations are obtained by applying Kirchhoff's laws and allowing the length of the line to approach zero. The voltage and current waves on the line are

$$V(z, t) = V_o^+ e^{-\alpha z} \cos(\omega t - \beta z) + V_o^- e^{\alpha z} \cos(\omega t + \beta z)$$

$$I(z, t) = \frac{V_o^+}{Z_o} e^{-\alpha z} \cos(\omega t - \beta z) - \frac{V_o^-}{Z_o} e^{\alpha z} \cos(\omega t + \beta z)$$

showing that there are two waves traveling in opposite directions on the line.

3. The characteristic impedance Z_o (analogous to the intrinsic impedance η of plane waves in a medium) of a line is given by

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

and the propagation constant γ (in per meter) is given by

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

The wavelength and wave velocity are

$$\lambda = \frac{2\pi}{\beta}, \qquad u = \frac{\omega}{\beta} = f\lambda$$

4. The general case is that of the lossy transmission line ($G \neq 0 \neq R$) considered earlier. For a lossless line, $R = 0 = G$; for a distortionless line, $R/L = G/C$. It is desirable that power lines be lossless and telephone lines be distortionless.

5. The voltage reflection coefficient at the load end is defined as

$$\Gamma_L = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

and the standing wave ratio is

$$s = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

where Z_L is the load impedance.

6. At any point on the line, the ratio of the phasor voltage to phasor current is the impedance at that point looking towards the load and would be the input impedance to the line if the line were that long. For a lossy line,

$$Z(z) = \frac{V_s(z)}{I_s(z)} = Z_{in} = Z_o \left[\frac{Z_L + Z_o \tanh \gamma \ell}{Z_o + Z_L \tanh \gamma \ell}\right]$$

where ℓ is the distance from load to the point. For a lossless line ($\alpha = 0$), $\tanh \gamma\ell = j \tan \beta\ell$; for a shorted line, $Z_L = 0$; for an open-circuited line, $Z_L = \infty$; and for a matched line, $Z_L = Z_o$.

7. The Smith chart is a graphical means of obtaining line characteristics such as Γ, s, and Z_{in}. It is constructed within a circle of unit radius and based on the formula for Γ_L given above. For each r and x, it has two explicit circles (the resistance and reactance circles) and one implicit circle (the constant s-circle). It is conveniently used in determining the location of a stub tuner and its length. It is also used with the slotted line to determine the value of the unknown load impedance.
8. When a dc voltage is suddenly applied at the sending end of a line, a pulse moves forth and back on the line. The transient behavior is conveniently analyzed using the bounce diagrams.
9. Microstrip transmission lines are useful in microwave integrated circuits. Useful formulas for constructing microstrip lines and determining losses on the lines have been presented.

REVIEW QUESTIONS

11.1 Which of the following statements are not true of the line parameters R, L, G, and C?

(a) R and L are series elements.
(b) G and C are shunt elements.
(c) $G = \dfrac{1}{R}$.
(d) $LC = \mu\varepsilon$ and $RG = \sigma\varepsilon$.
(e) Both R and G depend on the conductivity of the conductors forming the line.
(f) Only R depends explicitly on frequency.
(g) The parameters are not lumped but distributed.

11.2 For a lossy transmission line, the characteristic impedance does not depend on

(a) The operating frequency of the line
(b) The length of the line
(c) The load terminating the line
(d) The conductivity of the conductors
(e) The conductivity of the dielectric separating the conductors

11.3 Which of the following conditions will not guarantee a distortionless transmission line?

(a) $R = 0 = G$
(b) $RC = GL$
(c) Very low frequency range ($R \gg \omega L$, $G \gg \omega C$)
(d) Very high frequency range ($R \ll \omega L$, $G \ll \omega C$)

11.4 Which of these is not true of a lossless line?

(a) $Z_{\text{in}} = -jZ_o$ for a shorted line with $\ell = \lambda/8$.
(b) $Z_{\text{in}} = j\infty$ for a shorted line with $\ell = \lambda/4$.

(c) $Z_{in} = jZ_o$ for an open line with $\ell = \lambda/2$.

(d) $Z_{in} = Z_o$ for a matched line.

(e) At a half-wavelength from a load, $Z_{in} = Z_L$ and repeats for every half-wavelength thereafter.

11.5 A lossless transmission line of length 50 cm with $L = 10\ \mu$H/m, $C = 40$ pF/m is operated at 30 MHz. Its electrical length is

(a) 20λ

(b) 0.2λ

(c) $108°$

(d) 40π

(e) None of the above

11.6 Match the following normalized impedances with points A, B, C, D, and E on the Smith chart of Figure 11.40.

(i) $0 + j0$ (ii) $1 + j0$

(iii) $0 - j1$ (iv) $0 + j1$

(v) $\infty + j\infty$ (vi) $\left[\frac{Z_{in}}{Z_o}\right]_{min}$

(vii) $\left[\frac{Z_{in}}{Z_o}\right]_{max}$ (viii) Matched load ($\Gamma = 0$)

11.7 A 500-m lossless transmission line is terminated by a load which is located at P on the Smith chart of Figure 11.41. If $\lambda = 150$ m, how many voltage maxima exist on the line?

(a) 7

(b) 6

(c) 5

(d) 3

(e) None

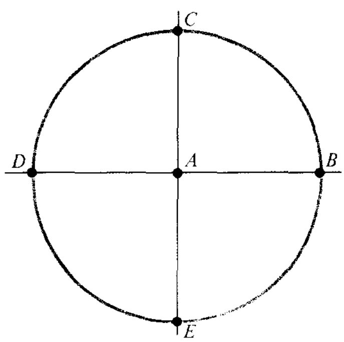


Figure 11.40 For Review Question 11.6.

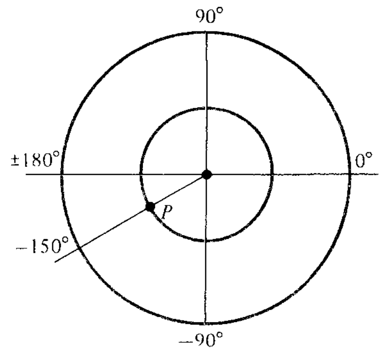


Figure 11.41 For Review Question 11.7.

11.8 Write true (T) or false (F) for each of the following statements.

(a) All r- and x-circles pass through point $(\Gamma_r, \Gamma_i) = (1, 0)$.
(b) Any impedance repeats itself every $\lambda/4$ on the Smith chart.
(c) An $s = 2$ circle is the same as $|\Gamma| = 0.5$ circle on the Smith chart.
(d) The basic principle of any matching scheme is to eliminate the reflected wave between the source and the matching device.
(e) The slotted line is used to determine Z_L only.
(f) At any point on a transmission line, the current reflection coefficient is the reciprocal of the voltage reflection coefficient at that point.

11.9 In an air line, adjacent maxima are found at 12.5 cm and 37.5 cm. The operating frequency is

(a) 1.5 GHz
(b) 600 MHz
(c) 300 MHz
(d) 1.2 GHz

11.10 Two identical pulses each of magnitude 12 V and width 2 μs are incident at $t = 0$ on a lossless transmission line of length 400 m terminated with a load. If the two pulses are separated 3 μs (similar to the case of Figure 11.53) and $u = 2 \times 10^8$ m/s, when does the contribution to $V_L(\ell, t)$ by the second pulse start overlapping that of the first?

(a) $t = 0.5\ \mu$s
(b) $t = 2\ \mu$s
(c) $t = 5\ \mu$s
(d) $t = 5.5\ \mu$s
(e) $t = 6\ \mu$s

Answers: 11.1c,d,e, 11.2b,c, 11.3c, 11.4a,c, 11.5c, 11.6 (i) D,B, (ii) A, (iii) E, (iv) C, (v) B, (vi) D, (vii) B, (viii) A, 11.7a, 11.8 (a) T, (b) F, (c) F, (d) T, (e) F, (f) F, 11.9b, 11.10e.

PROBLEMS

11.1 An air-filled planar line with $w = 30$ cm, $d = 1.2$ cm, $t = 3$ mm has conducting plates with $\sigma_c = 7 \times 10^7$ S/m. Calculate R, L, C, and G at 500 MHz.

11.2 The copper leads of a diode are 16 mm in length and have a radius of 0.3 mm. They are separated by a distance of 2 mm as shown in Figure 11.42. Find the capacitance between the leads and the ac resistance at 10 MHz.

***11.3** In Section 11.3, it was mentioned that the equivalent circuit of Figure 11.5 is not the only possible one. Show that eqs. (11.4) and (11.6) would remain the same if the Π-type and T-type equivalent circuits shown in Figure 11.43 were used.

11.4 A 78-Ω lossless planar line was designed but did not meet a requirement. What fraction of the widths of the strip should be added or removed to get the characteristic impedance of 75 Ω?

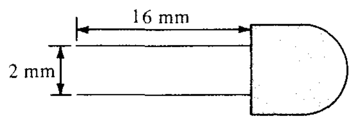


Figure 11.42 The diode of Problem 11.2.

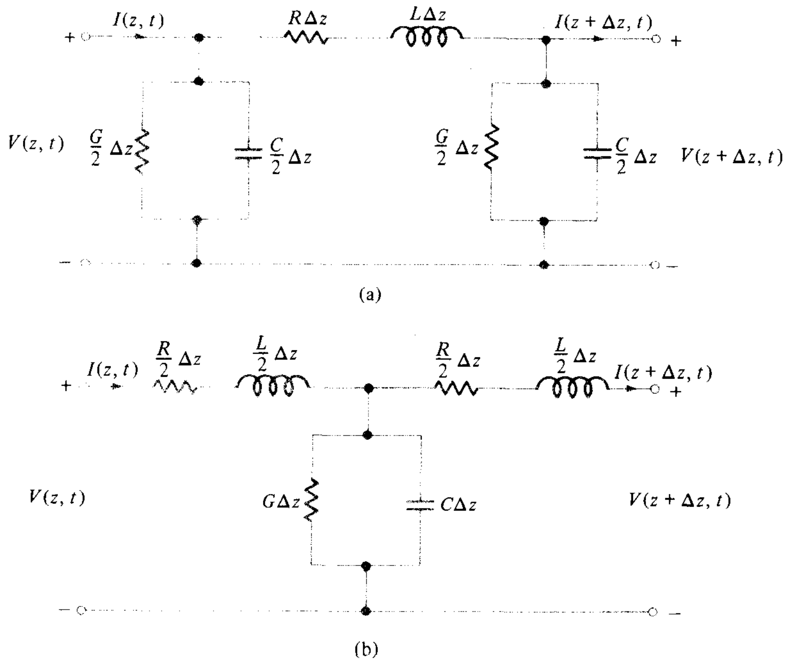


Figure 11.43 For Problem 11.3: **(a)** Π-type equivalent circuit, **(b)** T-type equivalent circuit.

11.5 A telephone line has the following parameters:

$$R = 40\ \Omega/\text{m}, \qquad G = 400\ \mu\text{S/m}, \qquad L = 0.2\ \mu\text{H/m}, \qquad C = 0.5\ \text{nF/m}$$

(a) If the line operates at 10 MHz, calculate the characteristic impedance Z_o and velocity u. (b) After how many meters will the voltage drop by 30 dB in the line?

11.6 A distortionless line operating at 120 MHz has $R = 20\ \Omega/\text{m}$, $L = 0.3\ \mu\text{H/m}$, and $C = 63$ pF/m. (a) Determine γ, u, and Z_o. (b) How far will a voltage wave travel before it is reduced to 20% of its initial magnitude? (c) How far will it travel to suffer a 45° phase shift?

11.7 For a lossless two-wire transmission line, show that

(a) The phase velocity $u = c = \dfrac{1}{\sqrt{LC}}$

(b) The characteristic impedance $Z_o = \dfrac{120}{\sqrt{\varepsilon_r}} \cosh^{-1} \dfrac{d}{2a}$

Is part (a) true of other lossless lines?

11.8 A twisted line which may be approximated by a two-wire line is very useful in the telephone industry. Consider a line comprised of two copper wires of diameter 0.12 cm that have a 0.32-cm center-to-center spacing. If the wires are separated by a dielectric material with $\varepsilon = 3.5\varepsilon_o$, find L, C, and Z_o.

11.9 A lossless line has a voltage wave

$$V(z, t) = V_o \sin(\omega t - \beta z)$$

Find the corresponding current wave.

11.10 On a distortionless line, the voltage wave is given by

$$V(\ell') = 120e^{0.0025\ell'} \cos(10^8 t + 2\ell') + 60e^{-0.0025\ell'} \cos(10^8 t - 2\ell')$$

where ℓ' is the distance from the load. If $Z_L = 300\ \Omega$, find: (a) α, β, and u, (b) Z_o and $I(\ell')$.

11.11 (a) Show that a transmission coefficient may be defined as

$$\tau_L = \frac{V_L}{V_o^+} = 1 + \Gamma_L = \frac{2Z_L}{Z_L + Z_o}$$

(b) Find τ_L when the line is terminated by: (i) a load whose value is nZ_o, (ii) an open circuit, (iii) a short circuit, (iv) $Z_L = Z_o$ (matched line).

11.12 A coaxial line 5.6 m long has distributed parameters $R = 6.5\ \Omega/\text{m}$, $L = 3.4\ \mu\text{H/m}$, $G = 8.4$ mS/m, and $C = 21.5$ pF/m. If the line operates at 2 MHz, calculate the characteristic impedance and the end-to-end propagation time delay

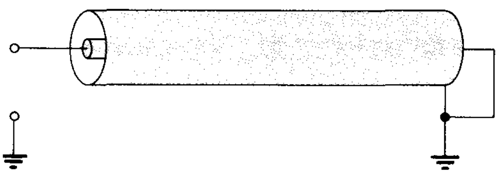


Figure 11.44 For Problem 11.16.

11.13 A lossless transmission line operating at 4.5 GHz has $L = 2.4\ \mu$H/m and $Z_o = 85\ \Omega$. Calculate the phase constant β and the phase velocity u.

11.14 A 50-Ω coaxial cable feeds a $75 + j20$-Ω dipole antenna. Find Γ and s.

11.15 Show that a lossy transmission line of length ℓ has an input impedance $Z_{sc} = Z_o \tanh \gamma\ell$ when shorted and $Z_{oc} = Z_o \coth \gamma\ell$ when open. Confirm eqs. (11.37) and (11.39).

11.16 Find the input impedance of a short-circuited coaxial transmission line of Figure 11.44 if $Z_o = 65 + j38\ \Omega$, $\gamma = 0.7 + j2.5$ /m, $\ell = 0.8$ m.

11.17 Refer to the lossless transmission line shown in Figure 11.45. (a) Find Γ and s. (b) Determine Z_{in} at the generator.

11.18 A quarter-wave lossless 100-Ω line is terminated by a load $Z_L = 210\ \Omega$. If the voltage at the receiving end is 80 V, what is the voltage at the sending end?

11.19 A 500-Ω lossless line has $V_L = 10e^{j25°}$ V, $Z_L = 50e^{j30°}$. Find the current at $\lambda/8$ from the load.

11.20 A 60-Ω lossless line is connected to a source with $V_g = 10\underline{/0°}\ V_{rms}$ and $Z_g = 50 - j40\ \Omega$ and terminated with a load $j40\ \Omega$. If the line is 100 m long and $\beta = 0.25$ rad/m, calculate Z_{in} and V at

(a) The sending end
(b) The receiving end
(c) 4 m from the load
(d) 3 m from the source

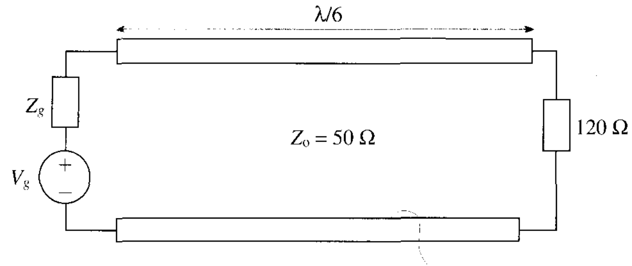


Figure 11.45 For Problem 11.17.

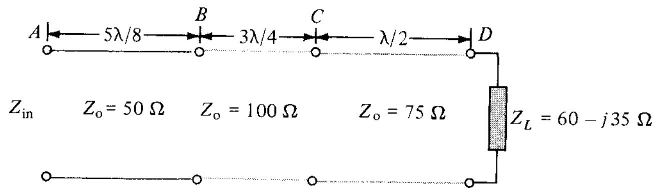


Figure 11.46 For Problem 11.22.

11.21 A lossless transmission line with a characteristic impedance of 75 Ω is terminated by a load of 120 Ω. The length of the line is 1.25λ. If the line is energized by a source of 100 V (rms) with an internal impedance of 50 Ω, determine: (a) the input impedance, and (b) the magnitude of the load voltage.

***11.22** Three lossless lines are connected as shown in Figure 11.46. Determine Z_{in}.

***11.23** Consider the two-port network shown in Figure 11.47(a). The relation between the input and output variables can be written in matrix form as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

For the lossy line in Figure 11.47(b), show that the *ABCD* matrix is

$$\begin{bmatrix} \cosh \gamma \ell & Z_o \sinh \gamma \ell \\ \dfrac{1}{Z_o} \sinh \gamma \ell & \cosh \gamma \ell \end{bmatrix}$$

11.24 A 50-Ω lossless line is 4.2 m long. At the operating frequency of 300 MHz, the input impedance at the middle of the line is $80 - j60\ \Omega$. Find the input impedance at the generator and the voltage reflection coefficient at the load. Take $u = 0.8c$.

11.25 A 60-Ω air line operating at 20 MHz is 10 m long. If the input impedance is $90 + j150\ \Omega$, calculate Z_L, Γ, and s.

11.26 A 75-Ω transmission line is terminated by a load of $120 + j80\ \Omega$. (a) Find Γ and s. (b) Determine how far from the load is the input impedance purely resistive.

11.27 A 75-Ω transmission line is terminated by a load impedance Z_L. If the line is 5λ/8 long, calculate Z_{in} when: (a) $Z_L = j45\ \Omega$, (b) $Z_L = 25 - j65$.

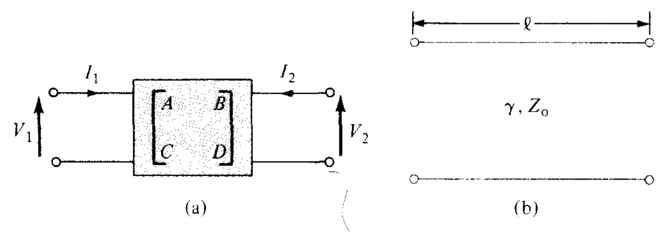


Figure 11.47 For Problem 11.23.

11.28 Determine the normalized input impedance at $\lambda/8$ from the load if: (a) its normalized impedance is $2 + j$, (b) its normalized admittance is $0.2 - j0.5$, (c) the reflection coefficient at the load is $0.3 + j0.4$.

11.29 A transmission line is terminated by a load with admittance $Y_L = (0.6 + j0.8)/Z_o$. Find the normalized input impedance at $\lambda/6$ from the load.

11.30 An 80-Ω transmission line operating at 12 MHz is terminated by a load Z_L. At 22 m from the load, the input impedance is $100 - j120\ \Omega$. If $u = 0.8c$,

(a) Calculate Γ_L, $Z_{\text{in,max}}$, and $Z_{\text{in,min}}$.

(b) Find Z_L, s, and the input impedance at 28 m from the load.

(c) How many $Z_{\text{in,max}}$ and $Z_{\text{in,min}}$ are there between the load and the $100 - j120\ \Omega$ input impedance?

11.31 An antenna, connected to a 150-Ω lossless line, produces a standing wave ratio of 2.6. If measurements indicate that voltage maxima are 120 cm apart and that the last maximum is 40 cm from the antenna, calculate

(a) The operating frequency

(b) The antenna impedance

(c) The reflection coefficient. Assume that $u = c$.

11.32 The observed standing-wave ratio on a 100-Ω lossless line is 8. If the first maximum voltage occurs at 0.3λ from the load, calculate the load impedance and the voltage reflection coefficient at the load.

11.33 A 50-Ω line is terminated to a load with an unknown impedance. The standing wave ratio $s = 2.4$ on the line and a voltage maximum occurs $\lambda/8$ from the load. (a) Determine the load impedance. (b) How far is the first minimum voltage from the load?

11.34 A 75-Ω lossless line is terminated by an unknown load impedance Z_L. If at a distance 0.2λ from the load the voltage is $V_s = 2 + j$ V while the current is 10 mA. Find Z_L and s.

11.35 Two $\lambda/4$ transformers in tandem are to connect a 50-Ω line to a 75-Ω load as in Figure 11.48.

(a) Determine the characteristic impedance Z_{o1} if $Z_{o2} = 30\ \Omega$ and there is no reflected wave to the left of A.

(b) If the best results are obtained when

$$\left[\frac{Z_o}{Z_{o1}}\right]^2 = \frac{Z_{o1}}{Z_{o2}} = \left[\frac{Z_{o2}}{Z_L}\right]^2$$

determine Z_{o1} and Z_{o2} for this case.

11.36 Two identical antennas, each with input impedance 74 Ω are fed with three identical 50-Ω quarter-wave lossless transmission lines as shown in Figure 11.49. Calculate the input impedance at the source end.

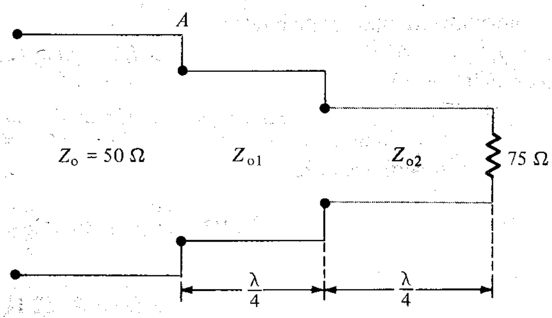


Figure 11.48 Double section transformer of Problem 11.35.

11.37 If the line in the previous problem is connected to a voltage source 120 V with internal impedance 80 Ω, calculate the average power delivered to either antenna.

11.38 Consider the three lossless lines in Figure 11.50. If $Z_o = 50\ \Omega$, calculate:

(a) Z_{in} looking into line 1
(b) Z_{in} looking into line 2
(c) Z_{in} looking into line 3

11.39 A section of lossless transmission line is shunted across the main line as in Figure 11.51. If $\ell_1 = \lambda/4$, $\ell_2 = \lambda/8$, and $\ell_3 = 7\lambda/8$, find y_{in_1}, y_{in_2}, and y_{in_3} given that $Z_o = 100\ \Omega$, $Z_L = 200 + j150\ \Omega$. Repeat the calculations if the shorted section were open.

11.40 It is desired to match a 50-Ω line to a load impedance of $60 - j50\ \Omega$. Design a 50-Ω stub that will achieve the match. Find the length of the line and how far it is from the load.

11.41 A stub of length 0.12λ is used to match a 60-Ω lossless line to a load. If the stub is located at 0.3λ from the load, calculate

(a) The load impedance Z_L
(b) The length of an alternative stub and its location with respect to the load
(c) The standing wave ratio between the stub and the load

11.42 On a lossless line, measurements indicate $s = 4.2$ with the first maximum voltage at $\lambda/4$ from the load. Determine how far from the load a short-circuited stub should be located and calculate its length.

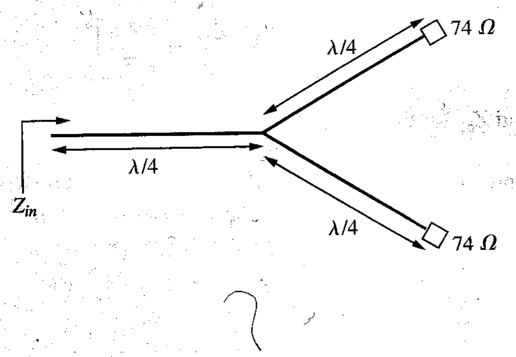


Figure 11.49 For Problems 11.36 and 11.37.

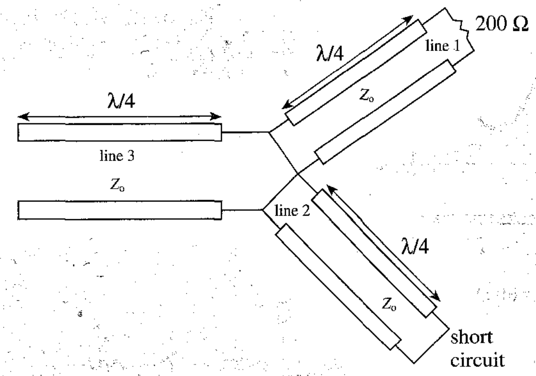


Figure 11.50 For Problem 11.38.

11.43 A 60-Ω lossless line terminated by load Z_L has a voltage wave as shown in Figure 11.52. Find s, Γ, and Z_L.

11.44 The following slotted-line measurements were taken on a 50-Ω system. With load: $s = 3.2$, adjacent $V_{\min}$ occurs at 12 cm and 32 cm (high numbers on the load side); with short circuit: $V_{\min}$ occurs at 21 cm. Find the operating frequency and the load impedance.

11.45 A 50-Ω air slotted line is applied in measuring a load impedance. Adjacent minima are found at 14 cm and 22.5 cm from the load when the unknown load is connected and $V_{\max} = 0.95$ V and $V_{\min} = 0.45$ V. When the load is replaced by a short circuit, the minima are 3.2 cm to the load. Determine s, f, Γ, and Z_L.

****11.46** Show that for a dc voltage V_g turned on at $t = 0$ (see Figure 11.30), the asymptotic values ($t \ll \ell/u$) of $V(\ell, t)$ and $I(\ell, t)$ are

$$V_\infty = \frac{V_g Z_L}{Z_g + Z_L} \quad \text{and} \quad I_\infty = \frac{V_g}{Z_g + Z_L}$$

11.47 A 60-Ω lossless line is connected to a 40-Ω pulse generator. The line is 6 m long and is terminated by a load of 100 Ω. If a rectangular pulse of width 5μ and magnitude 20 V is sent down the line, find $V(0, t)$ and $I(\ell, t)$ for $0 \leq t \leq 10\ \mu$s. Take $u = 3 \times 10^8$ m/s.

11.48 The switch in Figure 11.53 is closed at $t = 0$. Sketch the voltage and current at the right side of the switch for $0 < t < 6\ell/u$. Take $Z_o = 50\ \Omega$ and $\ell/u = 2\ \mu$s. Assume a lossless transmission line.

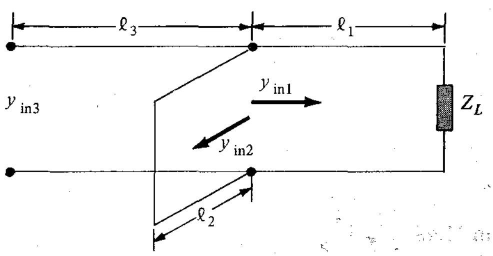


Figure 11.51 For Problem 11.39.

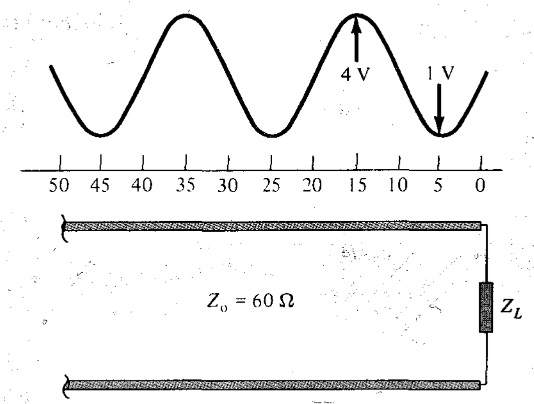


Figure 11.52 For Problem 11.43.

11.49 For the system shown in Figure 11.54, sketch $V(\ell, t)$ and $I(\ell, t)$ for $0 < t < 5\ \mu s$.

***11.50** Refer to Figure 11.55, where $Z_g = 25\ \Omega$, $Z_o = 50\ \Omega$, $Z_L = 150\ \Omega$, $\ell = 150$ m, $u = c$. If at $t = 0$, the pulse shown in Figure 11.56 is incident on the line

(a) Draw the voltage and current bounce diagrams.

(b) Determine $V(0, t)$, $V(\ell, t)$, $I(0, t)$, and $I(\ell, t)$ for $0 < t < 8\ \mu s$.

11.51 A microstrip line is 1 cm thick and 1.5 cm wide. The conducting strip is made of brass ($\sigma_c = 1.1 \times 10^7$ S/m) while the substrate is a dielectric material with $\varepsilon_r = 2.2$ and $\tan\theta = 0.002$. If the line operates at 2.5 GHz, find: (a) Z_o and ε_{eff}, (b) α_c and α_d, (c) the distance down the line before the wave drops by 20 dB.

11.52 A 50-Ω microstrip line has a phase shift of 45° at 8 GHz. If the substrate thickness is $h = 8$ mm with $\varepsilon_r = 4.6$, find: (a) the width of the conducting strip, (b) the length of the microstrip line.

11.53 An alumina substrate ($\varepsilon = 9.6\varepsilon_o$) of thickness 2 mm is used for the construction of a microstrip circuit. If the circuit designer has the choice of making the line width to be within 0.4 to 8.0 mm, what is the range of characteristic impedance of the line?

11.54 Design a 75-Ω microstrip line on a 1.2-mm thick-duroid ($\varepsilon_r = 2.3$) substrate. Find the width of the conducting strip and the phase velocity.

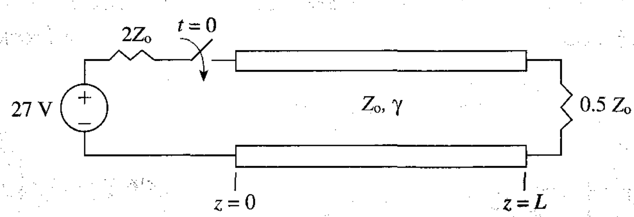


Figure 11.53 For Problem 11.48.

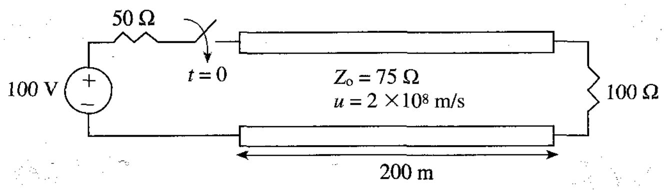


Figure 11.54 For Problem 11.49.

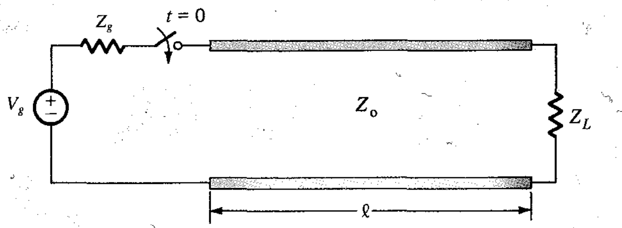


Figure 11.55 For Problem 11.50.

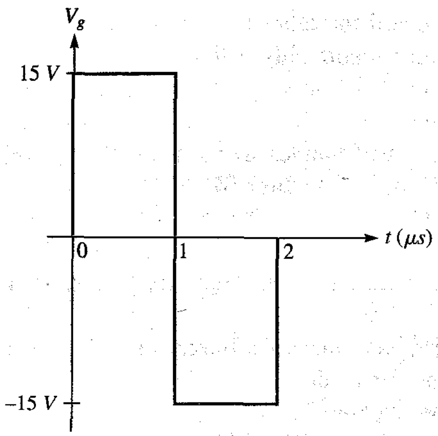


Figure 11.56 Two rectangular pulses of Problem 11.50.